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The art of Mathematics: Origami simulation

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ABSTRACT

In the real-world domain, mathematics is essential everywhere. Current technologies, like computer and information technology, can't also be developed without assisting mathematics. Mathematics includes the study of many fields, including number theory, algebra, geometry, mathematical analysis and so on. Unfortunately, mathematics is also influenced by the arts, like paper folding. It is also called "Origami". In this study, we will express the antiquity of origami and how to apply mathematics to making them. The exploration of the mathematics theory is an influence on the developing of Origami. The design of origamis can also be applied to the construction and industries. In this paper, we will discuss the simulation of the creation of origami.

Keywords— *Mathematics, Origami, Folding-art*

1. INTRODUCTION

Origami is an ancient Chinese and Japanese art of paper folding. From the Brief History of the Ancient Art of Paper folding, I gather that Origami gained acceptance in the West in the early 1950s [1]. In fact, origami can be used to explain many mathematical concepts in fields such as geometry, calculus, abstract algebra, and others. Origami originated in China where it was known as Zhe Zhi. It later became popular in Japan and is now considered a Japanese art [2].

In the geometry of paper folding, a straight line becomes a crease or a fold. Instead of drawing straight lines, one folds a piece of paper and flattens the crease. Folding paper is analogous to mirroring one half of a plane in a crease. Thus, folding means both drawing a crease and mapping one half of a plane onto another. As in the usual Geometry, the distinction is being made between experimentation with the physical paper and the abstract theory of "paper folding" [1].

The paper folding can apply to teach, and also can apply to the industrial designs. The paper folding designs can be applied by the basic mathematics formula. Moreover, there are many theories are used to develop the paper folding ways. For simulation purpose, we used "Origami Simulator - Amanda Ghassaei" [3], which can be available online on http://apps.amandaghassaei.com/OrigamiSimulator/. The remaining parts of the paper will present the theoretical background in section 2. Moreover, section 3 will explain some basic mathematical concepts which are used on origami. In section 4, we will express the simulation of crane (flapping bird). Finally, we will conclude our paper in section 5.

2. BACKGROUND THEORY

The origami design uses several mathematics ways, such as geometry, calculus, and algebra. The origin of origami is from China, but very popular in Japan and also known as Japanese art [2]. Origami is worth studying and exploring in other math-related fields. For example, there is a connection between origami and topology, even to graph theory, something that we don't usually assume origami would associate with [4].

There are many other countries are included in the origami antiquity. In the United State, John Blackman (1955), his pursuits are gardening, nature, and Ikebana which he merges with origami. In France, Vincent Floderer has moved away from conventional origami and has developed a whole new vocabulary of techniques, most famously crumpling. One of the origin, Japan, Akira Yoshizawa is widely considered the father of modern origami art. He devoted his life to his art, living in poverty as he perfected his craft and developed thousands of new designs [5].

There are many applications that are implementing with origami techniques. The research [6] also explained the formalization of fold that is generated from non-flat origami to perform origami geometric construction in 3D space. The other work [7] also proposed the usage of origami to construct a 3D spherical structure from 2D parylene-C film. The other work [8] presented a reconfigurable two-arm conical spiral antenna.

3. MATHEMATICS IN ORIGAMI

There are many techniques can be applied on paper folding design. The basic concept is based on the Pythagoras theorem. In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. The concept of Pythagoras is like in equation (1).

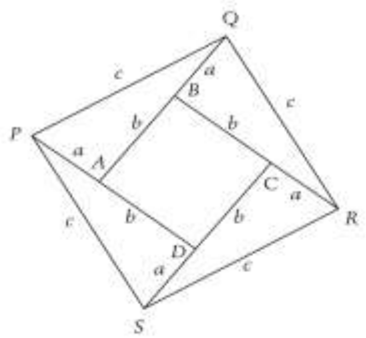

Fig. 1: The Pythagoras theorem applied to triangle APQ [10]

The concepts of length and area are discussed, stressing the fact that the Pythagorean Theorem is fundamentally a relationship between areas. It is useful to introduce units when discussing the length of a side of a triangle. This way, when the quantity is squared, it is possible to use the square footage of a house or apartment [9].

$$c^2 = a^2 + b^2 \quad (1)$$

Figure 1 shows one of the examples of applied Pythagoras on paper folding design. In figure, Square PQRS = Triangle PAQ + Triangle QBR + Triangle RCS + Triangle SDP + Square ABCD $= \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}ab + square\ ABCD \quad = 4 \times \frac{1}{2}ab + AB^2 = 2ab + (b-a)2, (Hence;\ c^2 = 2ab + b^2 + a^2 - 2ab)$.

Therefore, the result is as an equation 1 ($c^2 = a^2 + b^2$) according to the Pythagoras theorem.

4. SIMULATION

We used a simulator [3] to express which patterns are used in Origami. Allows you to simulate how any origami crease pattern will fold. It may look a little different from what you typically think of as "origami" - rather than folding paper in a set of sequential steps, this simulation attempts to fold every crease simultaneously. It does this by iteratively solving for small displacements in the geometry of an initially flat sheet due to forces exerted by creases [3].

We can apply the folding texture patterns on this simulator. The folding patterns are modeled as a pin-jointed framework, which allows the use of established structural engineering methods to gain insight into the kinematics of the folded sheet. The kinematic analysis can be naturally developed into a stiffness matrix approach; by studying its softest eigenmodes, important deformations of a partially folded sheet can be found, which aids in the understanding of Origami sheets for engineering applications [11].

We simulated the flapping bird, also seem as crane and the back view is shown in figure 2.

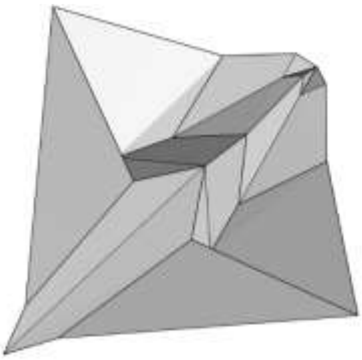

Fig. 2: Back view of the flapping bird

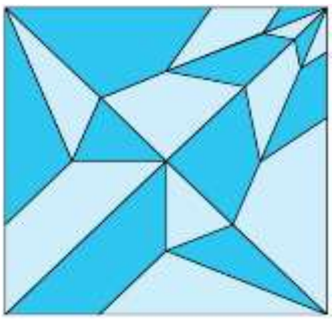

Fig. 3: The pattern of the flapping bird [12]

The pattern of the flapping pattern is shown in figure 3. And, the basic structure of the flapping bird is shown in figure 4.

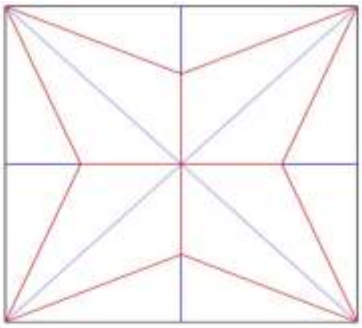

Fig. 4: Basic structure of flapping bird

The basic structure of the flapping bird can be applied by Kawasaki’s theorem [13]. Kawasaki's theorem is a theorem in the mathematics of paper folding that describes the crease patterns with a single vertex that may be folded to form a flat figure. It states that the pattern is flat-foldable if and only if alternatingly adding and subtracting the angles of consecutive folds around the vertex gives an alternating sum of zero. Crease patterns with more than one vertex do not obey such a simple criterion and are NP-hard to fold [14].

The work [12] explains that every flat-foldable origami crease pattern can be colored so that no 2 adjacent facets are the same color with only 2 colors which is shown in Figure 5. In Kawasaki condition, it alternates the angles around the vertex sum to the straight line.

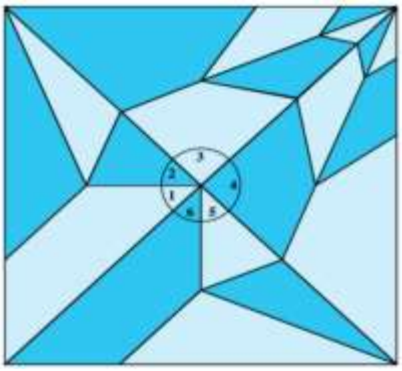

Fig. 5: Angle of the pattern [12]

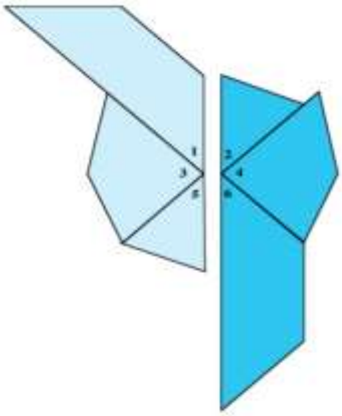

Fig. 6: Folding of the pattern [12]

In this theorem, it is a flat foldable as it is the crease pattern for a bird (crane). The origami is flag fold if the alternative sum of the consecutive angles is zero, which is shown in equation 2 [15].

$$a - b + c - d = 0 \quad (2)$$

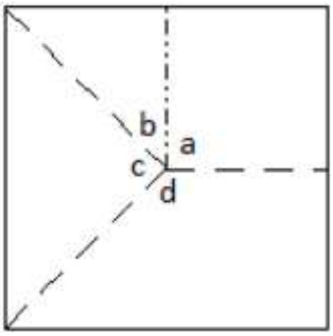

Fig. 7: The flat fold in origami [15]

In the Figure 7, a = 90°, b = 45°, c = 90°, d = 135 and the alternating sum is $a - b + c - d = 90° - 45° + 90° - 135° = 0$. It can be applied flat fold, Kawasaki's theorem.

5. CONCLUSION

Mathematics is very essential for everywhere. Every invention and creation need to be applied by mathematic theories. The paper folding, the art of mathematics, is also called origami, is invented on long time ago. But, the invention of the new techniques is still developing on this criterion. These folding techniques can also be applied to the industries. In this work, we simulated some origami design, especially on the creation of flapping bird. In future work, we will discuss new recent theories on folding art.

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