



## Economic and feasibility studies using an optimal control approach for enhanced oil recovery investment project

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### ABSTRACT

*Current harsh economic conditions, scarcity of natural resources and high market competition causes the emphasis on performing at higher efficiency rates. A feasibility study is a tool for investigating the viability of the prospective projects. This paper proposes an optimal control approach for an enhanced oil recovery investment project, to maximize the profit using the net present value (NPV) method. The present work discusses the operational, economic and financial feasibility studies used to assess the overall viability of the prospective project. The model is applied to a project being conducted by Egyptian petroleum research institute (EPRI) and the academy for scientific research and technology (ASRT). The total project budget is 20 million L.E. The costs are shared equally by EPRI and ASRT. The project financier is ASRT. The results showed that the proposed approach is accurate and given exact solutions.*

**Keywords**— Feasibility studies, NPV, Enhanced oil recovery, Semi-pilot, Steam-flooding, Optimal control, Closed loop optimal control, Petroleum, Gas projects

### 1. INTRODUCTION

Investments as one of the major economic issues raised and to create and sustain economic growth, capital formation in the country are very important [1]. The capital allocation process and the quality of its associated decisions remains a critical factor influencing overall performance [2]. The most common economic objectives for process optimization are profit and cost. These criteria have been applied to various problems, for example, a gross annual profit was optimized during reactive distillation optimal design [3]. NPV is the difference between the present value of cash inflows and the present value of cash outflows. NPV is used in capital budgeting to analyze the profitability of an investment or project. If the NPV of a prospective project is positive, it should be accepted. However, if NPV is negative, the project should probably be rejected because cash flows will also be negative [4]. The Issaran oilfield concession consists of 20,000 acres. It is located 290 km southeast of Cairo and 3 km inland from the western shore of the Gulf of Suez [5]. The Issaran field, a heavy to extra-heavy oil reservoir with reserves of approximately 500 MM bbl of oil, was discovered in 1981. The field primarily consists of three oil-bearing reservoirs ranging in depth from 1,000 to 2,000 ft. In the field, there are three formations: the Upper Dolomite, Lower Dolomite and Nukhul formations [6]. The heavy oil contained in fractured reservoirs represents a large portion of the total oil in place [7]. The reservoir characteristics of the Nukhul formation, which lies deeper than the Upper and Lower Dolomite formations, is quite different from the more shallow ones. It consists of a tight limestone matrix [6]. Issaran oilfield is one of the first fields in which steam EOR has been successfully implemented in a heavy oil carbonate reservoir [8]. Recovery methods used in heavy oil reservoirs usually yield very low recovery factors (5-10%). However, thermal recovery methods can be applied in order to enhance the heavy oil recovery. Steam-flooding is proven to be successful in light, heavy and extra heavy oil reservoirs. There are several mechanisms that govern the recovery of oil via steam-flooding. Some of the main mechanisms are thermal expansion, viscosity reduction, wettability alternation, steam distillation, and gas generation [7]. The design parameters of steam injection are vital to producing an economical project. In this paper the following parameters will be examined: steam to oil ratio (SOR), steam injection rate and steam quality.

The rest of the paper is organized as follows: in section 2, we describe the optimal control system. In section 3, we present the proposed optimal control approach. In section 4, we describe the thermally enhanced oil recovery project. Section 5 we present the results and discussion. Finally, section 6, presents the conclusion of the work.

### 2. OPTIMAL CONTROL SYSTEM

Optimal control System is a very useful mathematical field with many applications in both science and engineering. Recent years optimal Control system becomes a famous tool to solve problems of dynamic nature in management science and operations research [9]. [10] modeled dynamical systems by sets of differential equations. In order to state a discrete-time optimal control

problem over the periods  $0,1,2,\dots,T$ , we define the following [11]:  $\Theta$  is the set  $0,1,2,\dots,T-1$ ,  $Y^k$  is an  $n$ -component column state vector;  $k = 0,1,2,\dots,T$ ,  $u^k$  is an  $m$ -component column state vector;  $k = 0,1,2,\dots,T-1$ ,  $b^k$  is an  $s$ -component column state vector of constants;  $k = 0,1,2,\dots,T-1$ . The state  $Y^k$  is assumed to be measured at the beginning of period and control  $u^k$  is implemented during period  $k$ . We also define continuously differentiable functions  $f : E^n \times E^m \times \Theta \rightarrow E^n, F : E^n \times E^m \times \Theta \rightarrow E^1, g : E^m \times \Theta \rightarrow E^s$ , and  $S : E^m \times \Theta \{T\} \rightarrow E^1$ . Then, a discrete-time optimal control problem in the Bolza form is

$$\max \left\{ J = \sum_{k=0}^{T-1} F(x^k, u^k, k) + S(x^T, T) \right\} \tag{1}$$

Subject to the difference equation:

$$x^{k+1} - x^k = f(x^k, u^k, k), k = 0, \dots, T-1, x^0 \text{ given} \tag{2}$$

$$g(u^k, k) \geq b^k, k = 0, \dots, T-1 \tag{3}$$

**A Discrete Maximum Principle**

Applying the nonlinear programming theory to find the necessary conditions for the solution to the Mayer form of the control problem. We define the following [10]:  $\lambda^{k+1}$  is an  $n$ -component row vector of Lagrange multipliers,  $\mu^k$  is an  $s$ -component row vector of Lagrange multipliers associated with constraint (3). These multipliers are defined for each time  $k = 0, \dots, T-1$ . The Lagrangian function of the problem is

$$\mathcal{L} = \sum_{k=0}^{T-1} F(x^k, u^k, k) + S(x^T, T) + \sum_{k=0}^{T-1} \lambda^{k+1} [f(x^k, u^k, k) - x^{k+1} + x^k] + \sum_{k=0}^{T-1} \mu^k [g(u^k, k) - b^k] \tag{4}$$

We define the Hamiltonian function  $\mathcal{H}^k$  to be

$$\mathcal{H}^k = \mathcal{H}(x^k, u^k, k) = F(x^k, u^k, k) + \lambda^{k+1} f(x^k, u^k, k) \tag{5}$$

Using (5) we can rewrite (4) as

$$\mathcal{L} = S(x^T, T) + \sum_{k=0}^{T-1} [\mathcal{H}^k - \lambda^{k+1}(x^{k+1} - x^k)] + \sum_{k=0}^{T-1} \mu^k [g(u^k, k) - b^k] \tag{6}$$

If we differentiate (6) with respect to  $x^k$  for  $k = 1,2,\dots,T-1$ , we obtain

$$\Delta \lambda^k = \lambda^{k+1} - \lambda^k = -\frac{\partial \mathcal{H}^k}{\partial x^k}, k = 0,1,2,\dots,T-1 \tag{7}$$

If we differentiate (6) with respect to  $x^T$  we get

$$\frac{\partial \mathcal{L}}{\partial x^T} = \frac{\partial S}{\partial x^T} - \lambda^T = 0, \text{ or } \lambda^T = \frac{\partial S}{\partial x^T} \tag{8}$$

The difference equations (7) with terminal boundary conditions (8) are called the adjoint equations. If we differentiate  $\mathcal{L}$  with respect to  $u^k$  and state the corresponding Kuhn-Tucker conditions for the multiplier  $\mu^k$  and constraint (3), we have

$$\frac{\partial \mathcal{L}}{\partial u^k} = \frac{\partial \mathcal{H}^k}{\partial u^k} + \mu^k \frac{\partial g}{\partial u^k} = 0, \text{ or } \frac{\partial \mathcal{H}^k}{\partial u^k} = -\mu^k \frac{\partial g}{\partial u^k}, \tag{9}$$

and,

$$\mu^k \geq 0, \quad \mu^k [g(u^k, k) - b^k] = 0 \tag{10}$$

We note that, provided  $\mathcal{H}^k$  is concave in  $u^k, g(u^k, k)$  is concave in  $u^k$  and the constraint qualification holds, then conditions (9) and (10) are precisely the necessary and sufficient conditions for solving the following Hamiltonian maximization problem:

$$\begin{cases} \max_{u^k} \mathcal{H}^k \\ \text{Subject to} \\ g(u^k, k) \geq b^k. \end{cases} \tag{11}$$

**3. PROPOSED OPTIMAL CONTROL APPROACH**

**3.1. Notations**

The following assumptions are considered to build the model

- $i$  The interest rate
- $sv$  The salvage value
- $\alpha$  Tax rate
- $T$  Final time (length of time horizon)
- $Ecp$  Percentage of EPRI Credit
- $nt$  No. of trainer per year
- $pt$  US\$ Price per trainer
- $cr$  Changing rate from US\$ to L.E.
- $S1$  Study 1: Thermal EOR
- $S2$  Study 2: Chemical EOR
- $S3$  Study 3: Miscible EOR
- $SP1$  US\$ Study 1 Price
- $SP2$  US\$ Study 2 Price
- $SP3$  US\$ Study 3 Price
- $d$  No. of working days per year

<i>sp</i>	U\$ Steam Price
<i>m</i>	Cost of Maintenance per year
<i>P2</i>	The incentives -operation cost for managers and assistance team
<i>P3</i>	The supplies & materials- operation cost
<i>P4</i>	Spare parts -operation cost
<i>P5</i>	Others -operations cost
<i>P6</i>	Materials -operations cost
<i>Qt1</i>	Water consumption quantities
<i>Qt2</i>	Electricity consumption quantities
<i>sr</i>	Steam injected rate m <sup>3</sup> /d
<i>tc</i>	Training costs
<i>Wp</i>	Water price U\$/m <sup>3</sup>
<i>Ep</i>	Electricity price U\$/kWh
<i>eqp</i>	EQP Price
<i>I<sub>0</sub></i>	Initial investment
<i>Y1</i>	Expected number of study 1 per year
<i>Y2</i>	Expected number of study 2 per year
<i>Y3</i>	Expected number of study 3 per year
<i>Y4</i>	Incentives
<i>Y5</i>	Supplies and Materials
<i>Y6</i>	Spare Parts
<i>Y7</i>	Other Accessories
<i>Y8</i>	Maintenance
<i>Y9</i>	Materials
<i>Y10</i>	Water
<i>Y11</i>	Electricity
<i>Y12</i>	Equipment's and supplies & materials
<i>Y13</i>	Transportation, accommodation, and training
<i>u<sub>z</sub></i>	Desired rates of increasing, <i>z</i> = 0,1, ...,13

**3.2. The performance index**

The performance index of the model can be written as:

$$\begin{aligned} \max NPV = & \frac{1}{2} \cdot \frac{1}{(1-i)^{k-1}} \sum_{k=0}^{T-1} \left( (1-\alpha) \cdot \left( \left[ \begin{aligned} & (nt \cdot pt \cdot cr) + (S1.SP1.cr \cdot (y_1 - u_1)^2 \\ & + S2.SP2.cr \cdot (y_2 - u_2)^2 + S3.SP3.cr \cdot (y_3 - u_3)^2 \end{aligned} \right] \right. \right. \\ & - [ [d.sr.sp.cr + P2.(y_4 - u_4)^2 + P3.(y_5 - u_5)^2 + P4.(y_6 - u_6)^2 + P5.(y_7 - u_7)^2 + m.(y_8 - u_8)^2 + P6.(y_9 - u_9)^2 \\ & + Wp.cr.Q1.(y_{10} - u_{10})^2 + Ep.cr.Q2.(y_{11} - u_{11})^2 ] + tc.(y_{13} - u_{13})^2 ] \left. \right) + \alpha \cdot \left[ \frac{(1-sv) \cdot \left( \sum_{k=0}^{T-1} \frac{((y_{12} - u_{12})^2 * eqp)}{(1-i)^{t-1}} \right)}{T} \right] \left. \right) \\ & * Ecp \left. \right) - \left[ \left( \sum_{k=0}^{T-1} \frac{((y_{12} - u_{12})^2 * eqp)}{(1-i)^{t-1}} \right) \cdot \left( \frac{i \cdot (1+i)^t}{(1+i)^t - 1} \right) \right] + (sv) \cdot \left( \sum_{t=0}^{T-1} \frac{((y_{12} - u_{12})^2 * eqp)}{(1-i)^{k-1}} \right) \end{aligned} \quad (12)$$

Subject to

$$y^{k+1} - Y^k = f(y^k, u^k, k), k = 0, \dots, T - 1, Y^0 \text{ given} \quad (13)$$

$$y_j \geq 0, \quad j = 1, 2, \dots, 13 \quad (14)$$

**3.3. The mathematical model**

**Closed Loop Optimal Control**

The general form of linear quadratic regulator system [11] described by (15)

$$y(k + 1) = A(k)y(k) + B(k)u(k)$$

where  $k = k_0, k_1, \dots, k_{f-1}$ ,  $x(k)$  is  $n$ th order state vector,  $u(k)$  is  $r$ th order control vector, and  $A(k)$  and  $B(k)$  are matrices of  $n \times n$  and  $n \times r$  dimensions, respectively.

Given also a general performance index (PI) with terminal cost as

$$J = J(y(k_0), u(k_0), k_0) = \frac{1}{2} y'(k_f) F(k_f) x(k_f) + \frac{1}{2} \sum_{k=k_0}^{k_f-1} [y'(k) Q(k) x(k) + u'(k) R(k) u(k)] \quad (16)$$

We are given the boundary initial condition as

$$y(k = k_0) = x(k_0); x(k_f) \text{ is free, and } k_f \text{ is free} \quad (17)$$

where,  $F(k_f)$  and  $Q(k)$  are each  $n \times n$  order symmetric, positive semidefinite matrices, and  $R(k)$  is  $r \times r$  symmetric, positive definite matrix. This leads us to matrix difference Riccati equation (DRE) [10].

The solution of the equations (15) and (16) is as follows:

**Step 1:** Solve the matrix difference Riccati equation (DRE) [12]

$$P(k) = A'(k)P(k + 1)[I + E(k)P(k + 1)]^{-1}A(k) + Q(k) \tag{18}$$

With final condition

$$P(k = k_f) = F(k_f) \tag{19}$$

Where,

$$E(k) = B(k)R^{-1}(k)B'(k) \tag{20}$$

**Step 2:** Solve the optimal state  $x^*(k)$  from:

$$y^*(k + 1) = [A(k) - B(k)L(k)]y^*(k) \tag{21}$$

With initial condition

$$x(k_0) = x_0 \tag{22}$$

Where,

$$L(k) = R^{-1}(k)B'(k)A^{-T}(k)[P(k) - Q(k)] \tag{23}$$

**Step 3:** Obtain the optimal control  $u^*(k)$  from

$$u^*(k) = -L(k)y^*(k) \tag{24}$$

Where,  $L(k)$  is the Kalman gain.

**Step 4:** Obtain the optimal performance index from

$$J^* = \frac{1}{2}x^{*'}(k)P(k)y^*(k) \tag{25}$$

#### 4. THERMAL ENHANCED OIL RECOVERY PROJECT

The project title is "Constructing of semi-pilot plants for enhanced oil recovery by unconventional methods". The project is ongoing. The total project budget is 20 million L.E. The costs are shared equally by EPRI and the Academy for Scientific Research & Technology (ASRT). The ASRT is the project financier. The project duration is twenty-four months for the first construction phase. The project aims to link between industry and scientific research through the application of enhanced oil recovery technology by unconventional methods to recover oil at Egyptian oil fields. This goal requires the construction of a semi-pilot plant for enhanced oil recovery to test the suitability of these unconventional methods then pre-field applications and training of oil company engineers. This paper will apply the research to the Nukhul formation of the Issaran field. The project economic bases are described in [13].

#### 5. RESULTS AND DISCUSSION

The proposed model is used to solve the problem under discussion (EPRI project), where the parameters are listed in Table (1). All implementation made on a PC computer with a processor Intel CORE i7, and Matlab program version 2015a.

**Table 1: The vales of the parameters**

<b>Parameters</b>	$y_1(0)$	$y_2(0)$	$y_3(0)$	$y_4(0)$	$y_5(0)$	$y_6(0)$	$y_7(0)$
Values	0	0	0	2,000,000	0	0	500,000
<b>Parameters</b>	$y_8(0)$	$y_9(0)$	$y_{10}(0)$	$y_{11}(0)$	$y_{12}(0)$	$y_{13}(0)$	$u_1$
Values	0	0	0	0	17,000,000	500,000	
<b>Parameters</b>	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	$u_8$
Values	6	4	3	0.15	0.25	0.25	0.15
<b>Parameters</b>	$u_9$	$u_{10}$	$u_{11}$	$u_{12}$	$u_{13}$	Qt1	Qt2
Values	0.25	0.15	0.3	0.1	0.18	0.529	23
<b>Parameters</b>	Ecp	nt	P2	P3	P4	P5	P6
Values	0.75	30	0.10	0.25	0.15	0.05	0.10
<b>Parameters</b>	pt	SP1	SP2	SP3	tc		
Values	2000	50,000	30,000	25,000	396,000		

Table 2 summarizes the results of the proposed model at the time  $k = 7$ .

**Table 2: The Results of the proposed model at k=7**

<b>Parameters</b>	$y_1(7)$	$y_2(7)$	$y_3(7)$	$y_4(7)$	$y_5(7)$	$y_6(7)$	$y_7(7)$
Values	35	23	19	35,000,000	195,000	130,000	65,000
<b>Parameters</b>	$y_8(7)$	$y_9(7)$	$y_{10}(7)$	$y_{11}(7)$	$y_{12}(7)$	$y_{13}(7)$	NPV
Values	8,231,000	120,000	7.2	95	38,000,000	3,793,000	1350.5 M.L.E.

Table (3) summarizes the results of the financial feasibility study for the EPRI Project. The results of this study show that the project will be successful and has a great performance in all aspects. The results of the proposed optimal control model compared by the solutions for the same project in [13].

**Table 3: Comparison between the results for the EPRI Project (Laboratory Scenario) measures for the financial feasibility**

<i>Measures</i>	<i>Proposed Optimal Control Model</i>	<i>PSO [13]</i>
Total Investment (M L.E.)	1,967.3	1,886.5
NPV (M L.E.)	3,101.5	2,720.2
IRR (%)	33	29.32
Profitability Index	182.45	136.01
Pay Back Period (Years)	9	7
Net Profit (M L.E.)	1,652.67	1,462.02
ROI (%)	4.9	4.3

It is obvious from Table (3) that the financial feasibility measures are reflecting perfect performance for the project. The optimal investment levels increase from the optimum NPV. The profitability index and IRR are dependent on the quality of computing the NPV. ROI is a simple measure of economic performance based on cash flow, not profit and does not include a depreciation charge. In order to compare the performance of the considered financial feasibility study measures, the comparison between the solutions obtained in Table (3). The optimal total investment increase from solving the model by the proposed optimal control proposed approach solutions than the solutions obtained by solving the model by Particle Swarm Optimization (PSO).

## 6. CONCLUSION

The capital allocation process and the quality of its associated decisions remains a critical factor influencing overall performance. This research proposes a new optimal control approach to solve the economic and feasibility studies for the petroleum and gas projects problem. The proposed approach has described the principle of optimality to obtain the optimal solutions to the economic and feasibility studies for the petroleum and gas projects problem. The comparison between the solutions obtained by the approximation technique (PSO) and the proposed optimal control approach shown that the proposed approach is more accurate and given exact solutions. The given model may be extended in many ways.

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