



Design and Analysis of Spur Gear Train for APU Gear Box in a Manned Aircraft Vehicle

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ABSTRACT

An auxiliary power unit (APU) is a device on a vehicle that provides energy for functions other than propulsion. They are commonly found on large aircraft and naval ships as well as some large land vehicles. The primary purpose of an aircraft APU is to provide power to start the main engines. Turbine engines must be accelerated to a high rotational speed to provide sufficient air compression for self-sustaining operation. This work provides a good understanding of spur gear design and analysis from basics, considering bending, pitting and scoring criteria. Detailed discussion of various correction factors and overview of material selection for gears is covered. It also focuses on optimization and reliability. A solved example is provided for better understanding of the concepts. In this paper, we dealt with the whole procedure for design and analysis of spur gears into a nut shell. We assumed fundamental parameters and arrived at whole geometry. Based on the given data, assumed fundamental parameters like diametrical pitch, pressure angle, etc. and using tooth proportions we arrived at the geometry of tooth. Calculating loads and stresses for force analysis of individual gears in gear train and for calculating the loads and the stresses. We used stress equations suggested by AGMA. We Estimated strength or life of gears and for that, we need to assume a standard material for gears and estimate the strength of gears using strength equations suggested by AGMA. Later we evaluated Scoring resistance. Finally, we optimized and arrived at final design by performing several iterations for optimum design. In this way the different tasks were carried out and we arrived at a conclusion.

Keyword: Gear Train, APU, Gear Box, Spur Gear.

1. INTRODUCTION

The gearbox in the APU transfers power from the main shaft of the engine to an oil-cooled generator for electrical power. Within the gearbox, power is also transferred to engine accessories such as the fuel control unit, the lubrication module, and cooling fan. In addition, there is also a starter motor connected through the gear train to perform the starting function of the APU. Some APU designs use a combination starter/generator for APU starting and electrical power generation to reduce complexity.

2. PROCEDURE FOR DESIGN AND ANALYSIS

1. Assume fundamental parameters and arrive at whole geometry: Based on the given data, assume fundamental parameters like diametric pitch, pressure angle, etc. and using tooth proportions arrive at the geometry of tooth.
2. Calculate loads and stresses: Do force analysis of individual gears in gear train and calculate the loads. To calculate the stresses, use stress equations suggested by AGMA.
3. Estimate strength or the life of gears: Assume a standard material for gears and estimate the strength or life of gears using strength equations suggested by AGMA.
4. Evaluate Scoring resistance: Based on the discussion of scoring in the earlier section, evaluate the probability of scoring.
5. Optimize and arrive at final design: To arrive at a final design, do several iterations for optimum design.

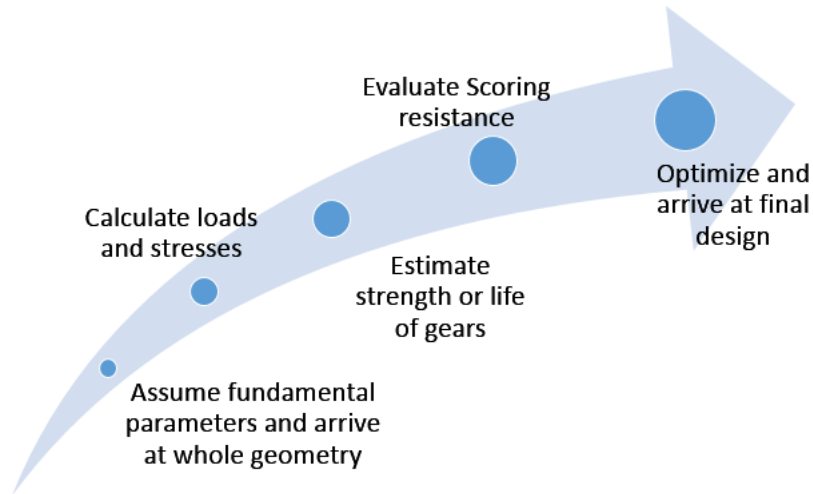


Fig-1: Gear Design Approach

3. OPTIMIZATION

This section deals with design optimization based on various geometry parameters, material, and lubricant oil. Several iterations are to be done to arrive at final design.

3.1 Preferred Number of Teeth

The following plot is generally applied to arrive at optimum number of teeth on pinion which considers failures like bending, pitting, scoring and also interference. The shaded zone is the safe zone for design.

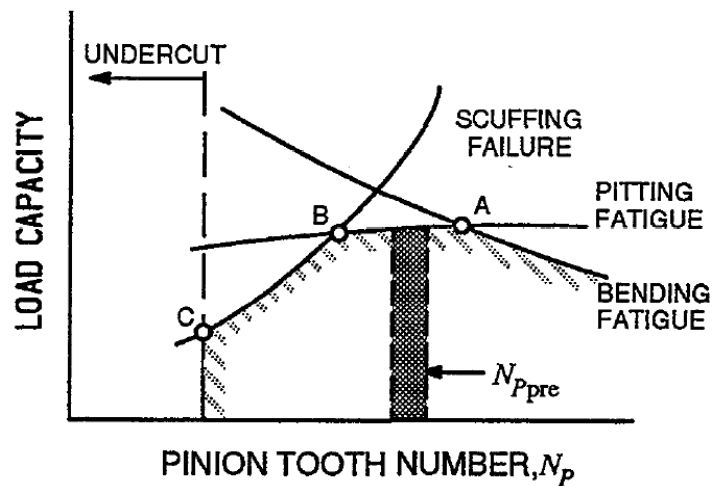


Fig-2: Preferred Number of Teeth on Pinion

3.2 Pressure Angle

The standard pressure angles in use are 20°, 22.5° and 25°:

Here is the comparison of tooth forms:

- ❑ 20° pressure angle
 - Most common for spur gears.
 - Thinner at the base.
 - Less load carrying capacity, when compared to 22.5° and 25° tooth.
- ❑ 22.5° pressure angle
 - Intermediate thickness at the base.
 - Can carry 11% more load than 20° tooth.
 - Less noisy operation.
- ❑ 25° pressure angle

- Thicker at base improves bending strength.
- Larger radii of curvature at pitch line.
- Can carry 20% more load than 20° tooth.
- More noise, since low contact ratio when not manufactured accurately.
- Thinner tips may lead to fracture at tips.

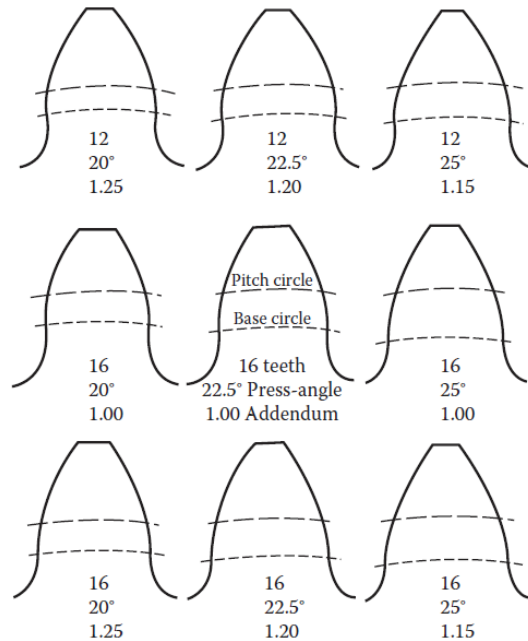


Fig-3: Comparison of Tooth Forms

3.3 Materials

Based on the strength or life requirement, select a proper material with appropriate heat treatment, hardness, and grade of material to minimize the weight and the cost.

3.4 Diametrical Tooth Forms

This parameter is also the geometry parameter, which could be any of the standard diametric pitches mentioned in the table below.

Diametric Pitch (1/in)	
Coarse	2, 2.25, 2.5, 3, 4, 6, 8, 10, 12, 16
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

3.5 Lubricant Oil

Selection of lubricant is based on the operating conditions like operating temperature and how much scoring resistance/probability is allowable. Proper lubricant leads to lesser scoring risk.

4. ANALYSIS

It is always desirable to enhance the level of understanding concepts with the help of relevant examples. Let us now solve a problem which is the title of this paper.

4.1 Problem Statement

Design spur gear train for Auxiliary Power Unit gear box considering resistance to Bending, Pitting and Scoring and meeting life requirements.

4.2 Inputs

Gear box consists of four spur gears. Load conditions, speeds, and required life are given as input data.

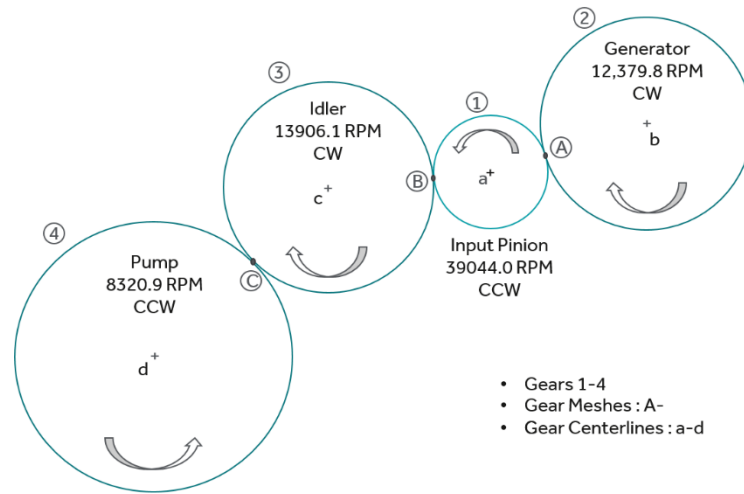


Fig-4: Gear Train

Table-1: Input Values

Accessory Location	Load Condition	Units	Shaft Load	Min. Required Life* (Hrs)
Generator (2)	Max Continuous	hp	230.1	19983
	5-minute overload	hp	278.7	16.667
	5-second overload	hp	391.2	---
	Shear Section (max)	in-lb	4600	Single Cycle
Pump (4)	Normal	hp	12.5/9.32	67500
	Shear Section (max)	in-lb	1200	Single Cycle

*At Inlet Oil Temperature: 200°F

4.3 Gear Geometry Calculations

To arrive at the initial geometry, let's first assume basic parameters as follows:

- Diametral Pitch = 16 1/in
- Pressure angle = 22.5°
- Material = Steel AMS-6265

Table-2: Material Properties

Parameter	Description
Material	Steel-AMS 6265*
Heat Treatment	Carburized and surface hardened
Grade	2
Young’s Modulus of Elasticity (E)	2.90E07 psi
Poisson’s ratio (μ)	0.29
Allowable bending stress number (S_{at})	65 000 psi
Allowable contact stress number (S_{ac})	225 000 psi
Allowable yield strength number (S_{ay})	128 000 psi

Hence by using tooth proportions, the gear geometry is:

Table-3: Calculated Geometry of Gears

Description	Symbol	Unit	G1		G2	G3		G4
			Mesh A- Pinion	Mesh B- Pinion	Mesh A- Gear	Mesh B- Gear	Mesh C- Pinion	Mesh C- Gear
Minimum Face Width (given)	F_{min}	inch	1.175	1.175	1.095	0.2	0.2	0.2
Calculated face width	F	inch	0.982	0.982	0.982	0.589	0.589	0.589
Max Outer Circle Diameter	d_o	inch	1.656	1.656	4.954	4.424	4.414	7.294
Nominal Operating Pitch Circle Diameter	d	inch	1.531	1.531	4.829	4.299	4.289	7.169
Nominal Operating Pitch Circle Radius	r	inch	0.766	0.766	2.414	2.149	2.145	3.584
Min. Profile Contact Ratio	m_c		1.601	1.595	1.601	1.595	1.696	1.696
Whole Depth	h_r	inch	0.141	0.141	0.141	0.141	0.141	0.141
Circular Pitch	p	inch	0.196	0.196	0.196	0.196	0.196	0.196
Tooth Thickness	t	inch	0.098	0.098	0.098	0.098	0.098	0.098
Base Circle Diameter	d_b	inch	1.415	1.415	4.461	3.972	3.963	6.623
Base Pitch	p_b	inch	0.181	0.181	0.181	0.181	0.181	0.181

The contact ratio is more than 1.2, continuous contact is ensured.

4.4 Force Analysis

The force analysis is done for the entire gear train and free body diagram of each gear is drawn separately.

Table-4 Loads Notations and Representations

Notation	Representation
F	Resultant force
F^t	Tangential Force
F^r	Radial Force
T	Torque

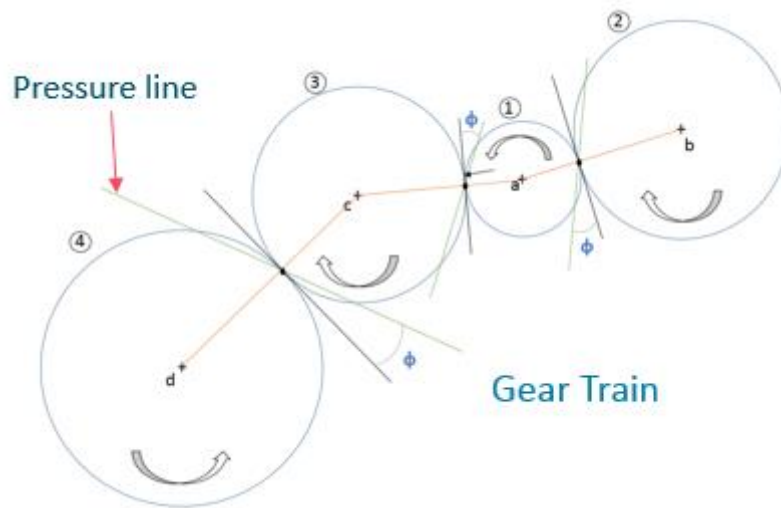


Fig-5: Force Analysis of Gear Train

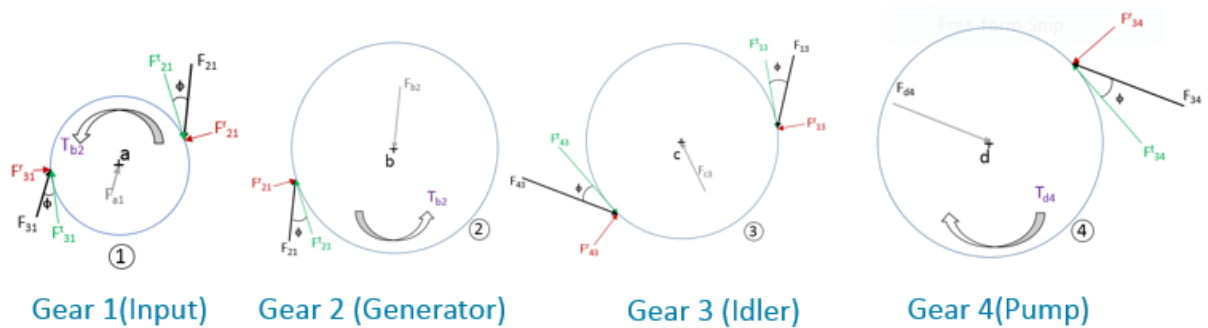


Fig-6: Free Body Diagrams of Individual Gears

Table 5 Calculation of Forces

Description	Symbol	Unit	G1		G2	G3		G4
			Mesh A-Pinion	Mesh B-Pinion	Mesh A-Gear	Mesh B-Gear	Mesh C-Pinion	Mesh C-Gear
Design Torque	T	in-lb	371.3	20.2	1171.0	56.6	56.6	94.6
Design Torque(5-Minute Overload)	T	in-lb	449.7	20.2	1418.4	56.6	56.6	94.6
Design Torque (5-Second Overload)	T	in-lb	631.3	20.2	1990.9	56.6	56.6	94.6
Design Tangential Load	W_t	lb	485.0	26.3	485.0	26.3	26.4	26.4
Design Tangential Load (5-Minute Overload)	W_t	lb	587.5	26.3	587.5	26.3	26.4	26.4
Design Tangential Load (5-Second Overload)	W_t	lb	824.6	26.3	824.6	26.3	26.4	26.4
Shear Section Overload	W_t	lb	1905.2	334.8	1905.2	334.8	334.8	334.8
Max Pitch line Velocity	V_t	ft/min	15,651	15,651	15,651	15,651	15,616	15,616

4.6 Correction Factors

Table-6: Calculated Loads and Pitch Line Velocity

Description	Symbol	G ₁ (Mesh A- Pinion)
Overload Factor	K _o	1.25
Dynamic Factor	K _v	1.11
Size Factor	K _s	1.00
Load Distribution Factor	K _m	1.20
Rim Thickness Factor	K _b	1.00
Geometry Factor for bending strength	J	0.32
Geometry factor for pitting resistance	I	0.12
Elastic Coefficient (lb/in ²) ^{0.5}	C _p	2241.32
Surface condition factor for pitting resistance	C _f	1.00
Temperature Factor	K _T	1.00
Reliability Factor	K _R	1.00
Hardness ratio Factor	C _H	1.00

4.7 Calculation of Stresses

Table-7: Typical Values for Various Correction Factors

Description	Symbol	Unit	G1		G2	G3		G4
			Mesh A- Pinion	Mesh B- Pinion	Mesh A- Gear	Mesh B-Gear	Mesh C- Pinion	Mesh C-Gear
Bending Stress (Max Continuous)	S _t	psi	34,364	1,867	30,908	3,103	3,732	3,180
Overload Bending Stress (5 Minutes)	S _t	psi	41,622	1,867	37,437	3,103	3,732	3,180
Overload Bending Stress (5 Seconds)	S _t	psi	58,423	1,867	52,548	3,103	3,732	3,180
Compressive Stress (Max Continuous)	S _c	psi	137,320	32,387	142,248	45,742	28,331	28,331
Compressive Stress (5-Minute Overload)	S _c	psi	151,128	32,387	156,552	45,742	28,331	28,331
Compressive Stress (5-Second Overload)	S _c	psi	179,051	32,387	185,476	45,742	28,331	28,331
Max Bending Stress (Shear Section Overload)	σ _b	psi	97,287	17,096	87,504	28,420	34,102	29,057
Margin of Safety (Bending)	(MOS) _b		0.42	7.07	0.58	3.86	3.05	3.75

4.8 Life Determination

Life based on Bending and Pitting is displayed.

Table-8: Life Based On Bending and Pitting

Description	Symbol	Unit	G1		G2	G3		G4
			Mesh A- Pinion	Mesh B- Pinion	Mesh A- Gear	Mesh B-Gear	Mesh C-Pinion	Mesh C-Gear
Estimated Life - Bending (Max Continuous)	L	hrs	1.59E+09		1.33E+11		5.58E+24	1.06E+29
Estimated Life - Bending (5-Minute Overload)	L	hrs	4.21E+06		3.53E+08		5.58E+24	1.06E+29
Estimated Life - Compressive (Max Continuous)	L	hrs	2.88E+04		4.84E+04		4.74E+05	9.69E+10
Estimated Life - Compressive (5-Minute Overload)	L	hrs	5.21E+03		8.75E+03		4.74E+05	9.69E+10

- Gear 1: Pulsating load but two pulses per cycle (Miner’s approach is used).
- Gear 3: Alternating (reverse cyclic) load. Mean stress factor of 0.7 is used for life estimation.
- Gears 2 and 4: Pulsating load

4.9 Prediction of scoring resistance

4.9.1 Flash Temperature Index Calculation

The Flash Temperature Index is calculated at all meshes at the outer tip.

Table-9: Calculation of Flash Temperature at Index at Outer Tip

Description	Symb ol	Unit	G ₁ (Mesh A- Pinion)	G ₁ (Mesh B- Pinion)	G ₂ (Mesh A- Gear)	G ₃ (Mesh B- Gear)	G ₃ (Mesh C- Pinion)	G ₄ (Mesh C- Gear)
Inlet oil Temperature	T _i	deg F	200	200	200	200	200	200
Gear Blank Temperature	T _b	deg F	220	220	220	220	220	220
Friction Coefficient	f		0.012	0.012	0.012	0.012	0.009	0.009
Absolute viscosity	μ _o	cP	4.14	4.14	4.14	4.14	4.14	4.14
Rolling Velocity	V	fps	146.72	146.72	116.33	118.23	118.01	110.92
Sliding velocity	V _s	fps	30.39	28.49	30.39	28.49	7.09	7.09
Sum velocity	V _t	fps	263.05	264.95	263.05	264.95	228.93	228.93
Sp. Loading	W	lbs./in.	412.78	22.42	442.93	44.73	44.83	44.83
Eff. Tangential Load	W _{te}	lbs.	485.01	26.35	4.81	0.29	26.41	0.12
Pinion Roll Angle at pitch dia	θ	radians	0.41	0.41	0.41	0.41	0.41	0.41
Roll Angle at Pinion limit dia	θ _{LD}	radians	0.23	0.23	0.23	0.23	0.23	0.23
Roll Angle at lowest point of single tooth contact on Pinion	θ _L	radians	0.35	0.35	0.43	0.44	0.40	0.42
Roll Angle at highest point of contact on Pinion	θ _H	radians	0.45	0.46	0.40	0.40	0.43	0.41
Roll angle at pinion O.D.	θ _o	radians	0.61	0.61	0.48	0.49	0.49	0.46
Load constant	K		1.00	1.00	0.01	0.01	1.00	0.00
Surface Finish, rms	S	micro in.	10	10	10	10	10	10
Radius of curvature at point of T _f calculation	ρ	in.	0.43	0.43	1.08	0.97	0.97	1.53
Scoring Geometry Factor	Z _t		0.032	0.030	0.030	0.029	0.011	0.011
Pressure angle at outer circle	φ	radians	0.547	0.547	0.450	0.456	0.456	0.432
Flash Temperature index	T_f	deg F	229.81	221.04	220.32	220.06	220.29	220.01

4.9.2 Minimum oil film thickness calculation

It is calculated at all meshes at outer tip

Table-10: Calculation of Film Thickness at Outer Tip

Description	Symb ol	Unit	G ₁ (Mesh A- Pinion)	G ₁ (Mesh B- Pinion)	G ₂ (Mesh A- Gear)	G ₃ (Mesh B- Gear)	G ₃ (Mesh C- Pinion)	G ₄ (Mesh C- Gear)
Lubricant factor	G		2659	2659	2659	2659	2659	2659
Absolute viscosity	μ _o	lb.- sec/in ²	6.00E-07	6.00E-07	6.00E-07	6.00E-07	6.00E-07	6.00E-07
Velocity factor	U		1.01E-10	1.04E-10	1.01E-10	1.04E-10	4.41E-11	4.41E-11
Loading factor	W _t '		5.49E-05	3.07E-06	5.89E-05	1.80E-05	7.93E-06	7.93E-06
Effective radius	R _x	in.	0.24	0.23	0.24	0.23	0.53	0.53
Lubrication velocity	u	i/s	1262.2	1264.2	1262.2	1264.2	1223.0	1223.0
Circle radius at Point of Film Thickness Calculation	r	in	0.828	0.828	2.477	2.212	2.207	3.647
Poisson's ratio	μ		0.290	0.290	0.290	0.290	0.290	0.290
Elastic modulus	E	lb./in ²	2.90E+07	2.90E+07	2.90E+07	2.90E+07	2.90E+07	2.90E+07
Aangular velocity	ω	rps	4088.7	4088.7	1296.4	1456.2	1456.2	871.4
Lubricant pressure-viscosity coefficient	α	in ² /lb	8.40E-05	8.40E-05	8.40E-05	8.40E-05	8.40E-05	8.40E-05
Effective elastic modulus for a gear set	E	lb./in ²	3.E+07	3.E+07	3.E+07	3.E+07	3.E+07	3.E+07
Minimum oil film thickness	h_{min}	micro in.	14.5	20.9	14.4	16.6	23.2	23.2
Surface roughness, rms	δ	micro in.	14.142	14.142	14.142	14.142	14.142	14.142
Film parameter	λ		1.03	1.48	1.02	1.18	1.64	1.64

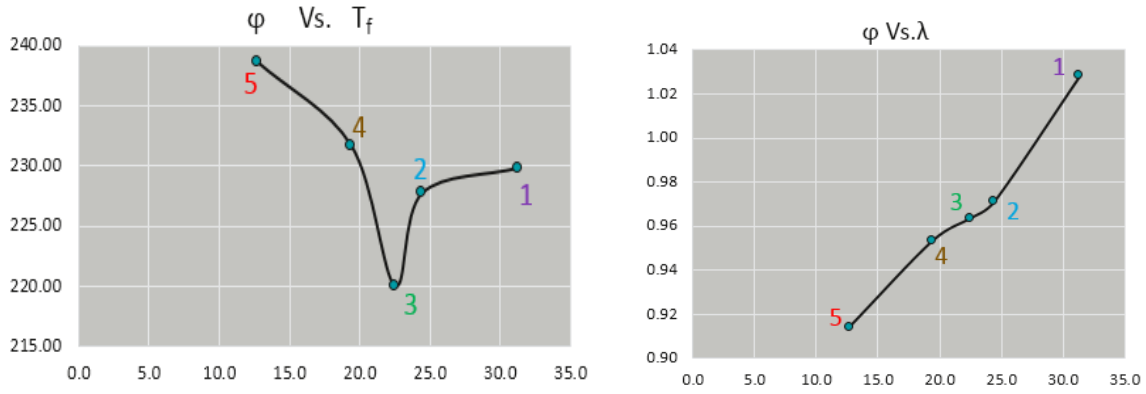


Fig-7: Variation of Flash Temperature index and Film parameter with pressure angle along the line of action

- Outside diameter
- Highest point of single tooth contact
- Pitch point
- Lowest point of single tooth contact
- Limit (contact) diameter

Table-11: Detailed Scoring Results

Gear-1 Mesh-A

S. No.	Location	Pressure angle (deg)	Roll Angle (deg)	Friction Coefficient	Flash Temperature Index (deg F)	Oil Film Thickness (micro in.)	Film Parameter
1	Outer Circle	31.3	34.9	0.012	229.81	14.5	1.03
2	HPSTC	24.4	26.0	0.019	227.72	13.7	0.97
3	Pitch Circle	22.5	23.7	0.202	220.00	13.6	0.96
4	LPSTC	19.4	20.2	0.018	231.64	13.5	0.95
5	Limit Circle	12.7	12.9	0.023	238.65	12.9	0.91

Gear-1 Mesh-B

S. No.	Location	Pressure angle (deg)	Roll Angle (deg)	Friction Coefficient	Flash Temperature Index (deg F)	Oil Film Thickness (micro in.)	Film Parameter
1	Outer Circle	31.3	34.9	0.012	221.04	20.9	1.48
2	HPSTC	24.5	26.1	0.003	220.14	19.7	1.39
3	Pitch Circle	22.5	23.7	0.194	220.00	19.5	1.38
4	LPSTC	19.4	20.2	0.002	220.13	19.3	1.37
5	Limit Circle	12.7	12.9	0.006	220.61	18.5	1.31

Gear-3 Mesh-C

S. No.	Location	Pressure angle (deg)	Roll Angle (deg)	Friction Coefficient	Flash Temperature Index (deg F)	Oil Film Thickness (micro in.)	Film Parameter
1	Outer Circle	26.1	28.1	0.009	220.29	23.2	1.64
2	HPSTC	23.1	24.5	0.013	220.31	22.7	1.60
3	Pitch Circle	22.5	23.7	0.123	220.00	22.6	1.60
4	LPSTC	21.8	22.9	0.012	220.34	22.6	1.60
5	Limit Circle	16.9	17.4	0.013	220.60	21.9	1.55

5. RESULTS

Table-12: Final Design Results

Parameters	Gear 1	Gear 2	Gear 3	Gear 4	Selection Based on
Pressure Angle (deg)	22.5	22.5	22.5	22.5	Advantages of Tooth Form
Material	AMS 6265	AMS 6265	AMS 6265	AMS 6265	Recommended by AGMA
Material Grade	2	2	1	1	Life Requirements
Material Heat Treatment	Carburized and surface hardened	Carburized and surface hardened	Through hardened	Through hardened	Life Requirements
Diametric Pitch (1/in)	16	16	16	16	Scoring
Surface Roughness (micro in)	10	10	10	10	Assumed based on study
Lubricant Oil	MIL – L - 23699	MIL – L - 23699	MIL – L - 23699	MIL – L - 23699	Scoring

6. CONCLUSIONS

The results have shown that the design is good enough to sustain required life based on both bending and pitting criteria. The calculated flash temperature index and the film parameter also signifies that the gears are quite resistant to scoring.

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