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A Study on Fuzzy Differential Equation Using Second Order Linear Equations

R. Ravichanthiran, N. Venkatesan
M. Phil, Research Scholar, Research Adviser
[12]*Department of Mathematics, Prist University, Thanjavur.*

ABSTRACT

The paper some results on boundary value problems for fuzzy differential equations with functional dependence" by J.J. Nieto and R. Rodriguez-Lopez addresses fuzzy and impulsive differential equations with various periodic boundary conditions. It is based on the Hukuhara derivative and makes strong use of explicit formulas for Green's functions. The article Generalized differentiability of fuzzy-valued numerical solutions.

Keywords: Differential Equations; Fuzzy Systems, Simpson's Method, Second Order Linear Equations.

INTRODUCTION

Fuzzy Differential Equations (FDEs) model have a wide range of applications in many branches of engineering and in the field of medicine. The concept of the fuzzy derivative was defined by Chang S.L. and Zadeh L.A. It was followed up by Dubosis. D and Prade. Who used extension principle in their approach? The term "fuzzy differential Equation" was introduced in1987 by Kandel. A and Byatt. W.J they have been many suggestions for the definition of fuzzy derivative to study "fuzzy differential Equation". In the literature, there are several approaches to study fuzzy differential equations. Under this interpretation, the solution of a fuzzy differential equation becomes less fuzzy as time goes on. The strong generalized derivative is defined for a large class of fuzzy number valued function than Hukuhara derivative. This case a fuzzy differential equation is not unique. it has two solutions locally. Recently some Mathematicians have studied fuzzy differential equations by Numerical methods. Numerical Solution of fuzzy differential equations has been introduced by M.Ma, M.Friedman, and Kandel. A in through Euler method. Taylor's method and Runge –Kutta methods have also been studied by authors.

Definition 1. A fuzzy number is a fuzzy set m:R→ $I = [0,1]$ with the properties:

- m is upper semi continuous,
- m(x) =0, outside of some interval [c,d],
- There are real number a and b, c$\leq a \leq b \leq d$ such that m is increasing on [c,a], decreasing on [b,d] and m(x)=1 for each xε[a,b].

We can identify a fuzzy number m with parameterized triples

m=(a(r).b(r)), r ε I

$$\text{Where } (a(r).b(r)) = \begin{cases} x/m(x) \geq r & if\ 0 < r \leq 1 \\ cl(\sup m) & if\ r = 0 \end{cases} \quad \text{--------------------------(1)}$$

Definition 2. We said that a fuzzy function X : [0, T] → F_cFC is a fuzzy solution of the fuzzy differential equation if it is G-differentiable on [0, T] and verifies the equation for all t ∈ [0, T].

Next we describe the procedure for solving the fuzzy differential equation

Problem 1. The second-order linear nonhomogeneous fuzzy differential equation

$$x''(t) = q_1 x'(t) + q_2 x(t) + A \text{ -------------------------------- } (2)$$

Where x,x' , A;[a,b]→ E^n are continuous function and q_1, q_2ε R\{0},with the initial conditions

$$X(t_0) = k_1,\ x'(t_0) = k_2, t_0 \varepsilon [a,b] \text{ -------------------------------}(3)$$

is equivalent to a Volterra-type fuzzy integral equation

$$x(t) = \int_{t_0}^{t} k(t, s, x(s))ds + g(t) \text{--------------------------------}(4)$$

Where k:[a,b]×[a,b] × E^n and g(t)=k_2(t-t_0) + k_1

The second-order linear fuzzy differential equation

$$x''(t) = q_1 x'(t) + q_2 x(t) + A(t), X(t_0) = k_1,\ x'(t_0) = k_2, t_0 \varepsilon [a,b] \text{----------------}(5)$$

is equivalent to the integral equation

$$x(t) = \int_{t_0}^{t} \left(\int_{t_0}^{t} f(t, s, x(s), x'(s))ds\right)ds + g(t)$$

Where f(t, x(t),$x'(t)$)= $q_1 x'(t)$+$q_2 x(t)$ +A (t) and g(t)=k_2(t-t_0) + k_1

let k:[a,b]×[a,b] × E^n → E^n be a map such that k(t, x(s))$-\int_{t_0}^{t} f(s, x(s), x'(s))ds$.

Then the above integral equation is equivalent to the integral equation (3).Thus, all the theorems on existence and uniqueness proved in hold true, equivalently for the fuzzy differential equation examined here.
Problem 2. The second-order linear nonhomogeneous differential equation

$$x''(t) = q_1 x(t) + A \text{ ------------------------------- } (6)$$

Where x,x', $A\varepsilon C([a,b], E^n)$and q_1ε R\{0}, with the initial conditions (3), has a unique solution $x\varepsilon C([a,b], E^n)$satisfying the initial conditions.
Let f:[a,b]]× $E^n \times E^n$ → E^nbe such that f(t, x(t),$x'(t)$)=$q_1 x(t)$ +A, tεT

Clearly, the map *f* is continuous and

$$D(f(t, x_1(t), x_2(t)), f(t, y_1(t), y_2(t)) = D(q_1 x_1(t) + A(t), (q_1 y_1(t) + A(t) \text{--------------}(7)$$

$$= D(q_1 x_1(t), q_1 y_1(t)) \leq |q_1| D(x_1(t), y_1(t)) \text{ for all } t\varepsilon [a,b], \text{----------}(8)$$

Thus, by the above problem has a unique solution.

Problem 3: Consider the following differential equation

$$u'(t) = u(1 - 2t),\ t \in [0,2] \text{-----------------------------}(9)$$

and the fuzzy initial value is given by:

$$u_0(s) = \begin{cases} 0 & if\ s < (-1/2) \\ 1 - 4s^2 & if\ (-1/2) \leq s \leq (1/2) \\ 0 & if\ s > (1/2) \end{cases} \text{ ---------------------------------- } (10)$$

The exact solution is

$$[u(t)]^{\alpha} = [(-\frac{\sqrt{1-\alpha}}{2})e^{-t(-1+t)}, (\frac{\sqrt{1-\alpha}}{2})]e^{-t(-1+t)}] \text{ --------------- } (11)$$

To get the approximate solution, we divide the interval [0, 2] into 20 equally spaced subintervals. Then, we proceed with the numerical method proposed in this paper. The obtained results are plotted. From the graph, we can see that the approximate solution converges to the exact solution. The local errors between the approximate and exact solutions at $t_N = 2$.

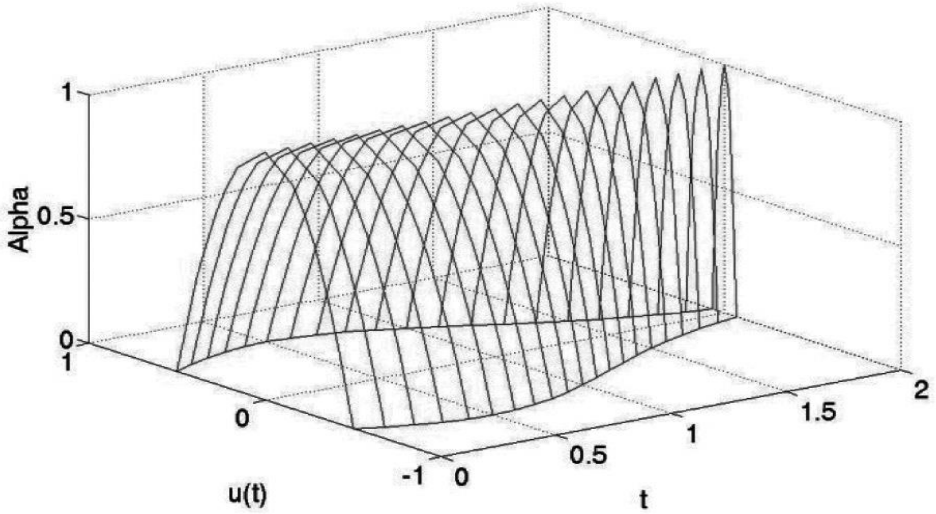

Figure1. The exact solution

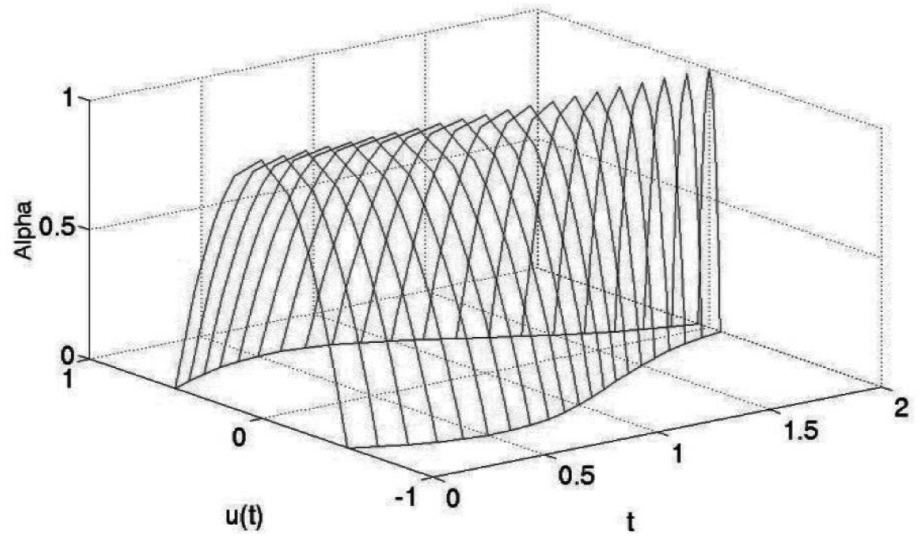

Figure 2. The approximate solution with h = 0.1 and N = 20

Exact solution (see Figure 3). It is diverging as *t* increases. This shows that the method proposed by Maetal. (1999) has overestimation in computation. This is always the case when we consider the same fuzzy interval as independent in fuzzy interval computations. Next, we study a non-linear differential equation with fuzzy initial value.

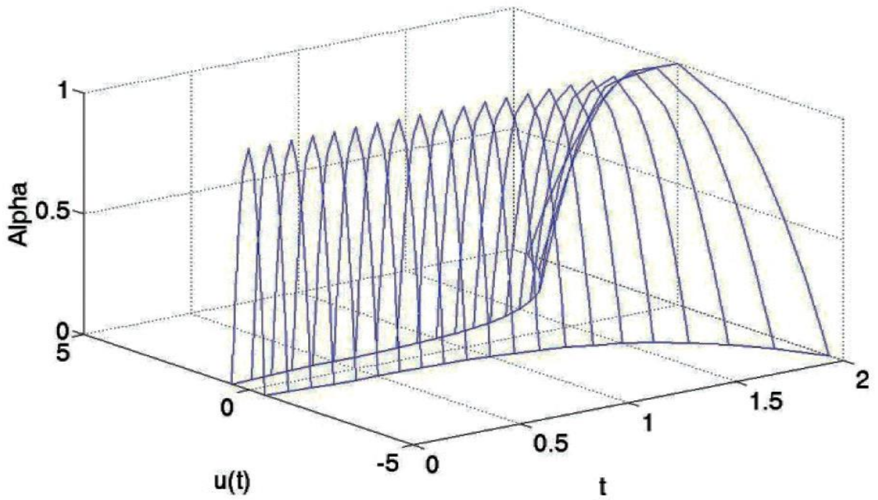

Figure 3. The approximate solution obtained by using the method proposed by Maetal. (1999) with *h* = 0.1 and *N* = 20

Numerical solution of fuzzy differential equations

In this section, discussion will be done for an interval $[t_{0,}T]$ of size h, the total error of the initial value problem by using the modified two-step Simpson's method is $[E]_r=[\underline{E}(r),\bar{E}(r)]$, where

$\underline{E}(r) = \frac{T-t_0)h^2}{6}\underline{g'}\,(\varepsilon_{2,}\, y(\varepsilon_{2,}))\,\underline{g_y}(t_{i+1},\varepsilon_3) - \frac{T-t_0)h^4}{90}\underline{g^4}\,\left(\varepsilon_{1,}\, y(\varepsilon_{1,})\right),$

$\bar{E}(r) = \frac{T-t_0)h^2}{6}\overline{g'}\,(\varepsilon_{2,}\, y(\varepsilon_{2,}))\,\overline{g_y}(t_{i+1},\varepsilon_3) - \frac{T-t_0)h^4}{90}\overline{g^4}\left(\varepsilon_{1,}\, y(\varepsilon_{1,})\right),$ $\qquad (\varepsilon_{1,}) = [\underline{(\varepsilon_{1,})}, \overline{(\varepsilon_{1,})}],$

$t_{i+1} \le \varepsilon_{1,} \le t_{i+1}$, $(\varepsilon_{2,}) = [\underline{(\varepsilon_{2,})}, \overline{(\varepsilon_{2,})}]$ $(\varepsilon_{2,})€(t_i, t_{i+1})$ and $(\varepsilon_{3,}) = [\underline{(\varepsilon_{3,})}, \overline{(\varepsilon_{3,})}$ is between $\underline{Y(t,r)}$+hF [t, $\underline{Y(t,r)}$, $\overline{Y(t,r)}$] and $\underline{Y(t,r)} + hF$ [t, $\underline{Y(t,r)}$, $\overline{Y(t,r)}$]+$\frac{h^2}{2}F'(\varepsilon_{2,}), y[\underline{(\varepsilon_{2,})}, \overline{y(\varepsilon_{2,})}]$ and it is in between $\overline{Y(t,r)}$]+hg[t, $\underline{Y(t,r)}$, $\overline{Y(t,r)}$] and$\overline{Y(t,r)}$] +hg [t, $\underline{Y(t,r)}$, $\overline{Y(t,r)}$]+ $\frac{h^2}{2}g'(\varepsilon_{2,}), y[\underline{(\varepsilon_{2,})}, \overline{y(\varepsilon_{2,})}]$

Let $\underline{E}(r)= \underline{R}(r)\,h^2 + \underline{S}(r)\,h^2$

$\underline{R}(r)\,h^2 = \frac{T-t_0)h^2}{6}\underline{g'}\,(\varepsilon_{2,}\, y(\varepsilon_{2,}))$, $\underline{S}(r)\,h^2=\frac{T-t_0)h^4}{90}\underline{g^4}\,\left(\varepsilon_{1,}\, y(\varepsilon_{1,})\right),$

May be chosen as a constant if

$\underline{g'}\,(\varepsilon_{2,}\, y(\varepsilon_{2,}))$, $\underline{g'}\,(t_{i+1},\varepsilon_{3,})$, and $\underline{g^4}\,\left(\varepsilon_{1,}\, y(\varepsilon_{1,})\right),$

Are reasonably constant. Suppose, evaluate

$\underline{I}(r) = \int_{t_{i=s}}^{t_{i+1}} Y'(s;r)ds$ by using the modified two-step Simpson's method with tree different subinterval$h_1, h_2 and\ h_3$. Let$L_1(r)$, $L_2(r)$and $L_3(r)$ be the approximations with errors $E_1(r)E_2(r)\ and\ E_3(r)$ respectively, then

L(r)= $L_1(r)$+ $E_1(r) - L_1(r) + R(r)h_1^2 + S(r)h_1^4$ ---------------------------(12)

L(r)= $L_2(r)$+ $E_2(r) - L_2(r) + R(r)h_2^2 + S(r)h_2^4$ --------------------------(13)

L(r)= $L_3(r)$+ $E_3(r) - L_3(r) + R(r)h_3^2 + S(r)h_3^4$ ---------------------------(14)

CONCLUSION

In this paper, we have studied the numerical solution of differential equations with fuzzy initial values. By taking into account the dependency problem in fuzzy computation, we proposed a new version of Simpson's method, which is a generalization of the conventional one. In order to show the capability of the proposed method, we conducted several numerical examples including linear and linear differential equations with fuzzy initial values. Final results showed that the numerical method proposed in this paper produced better solutions compared to the numerical method proposed in the literature.

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