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## Stabilization of Floating Body with Triple Pendulum

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### ABSTRACT

This paper presents an innovative method to stabilize any floating body caused by vibration due to sea waves, lateral movement, wind force and other causing forces with the use of a three-degree pendulum. Use of tuned mass dampers (TMD) is most common in high rise buildings, in ships or in bridges. In Single degree pendulum there is an only radial way to move and to counteract vibrations caused in any structure or floating bodies. Using three-degree pendulum in floating bodies like ships gives three degrees of freedom to pendulum to resist shocks occurring in this type of floating bodies. A parametric study is carried out to see the behaviour of this type of bodies without dampers, with single-degree pendulum and three-degree pendulum. The response of floating bodies to shocks was observed for varying conditions.

**Keywords:** Triple Pendulum, Vibration Control, Resistance against Shocks Waves, Behaviour of Pendulum, Damping, Tuned Mass Damper, Experimental Investigation.

### 1. INTRODUCTION

Vibration is a physical phenomenon that affects all floating bodies subjected to vibration due to sea waves, structures subjected to altering of loads etc. In luxury yacht building, there is a tendency towards a larger size. As a consequence, the structure becomes slender and hence flexible. In some cases, this tendency causes a hull girder natural frequency to coincide with propeller rotation frequencies, with annoying resonance as a consequence. The main issue is usually the vibration decay rate, which is very low. This indicates low structural damping, which is common in ship structures. Due to slight uncertainties in ship weights and structural damping, such a resonance can only be observed during sea trials. When it can be shown that the vibrations are indeed associated with a particular vibration mode, a Tuned Mass Damper (TMD) is a viable option to deal with the phenomenon. The waves can cause not only comfort problems but also safety problems. <sup>[1]</sup> To contract shock in systems there are mainly two types of controllers active and passive. Active controllers require a large amount of power source to operate. Passive control devices do not require an external power source to operate. Mostly passive dampers are being used in structures like hysteretic metallic dampers, frictional dampers, viscoelastic dampers, viscous fluid dampers, tuned mass damper and damping liquid. <sup>[2]</sup> In this research paper three-degree pendulum is being introduced as damping device for floating bodies like big ships.

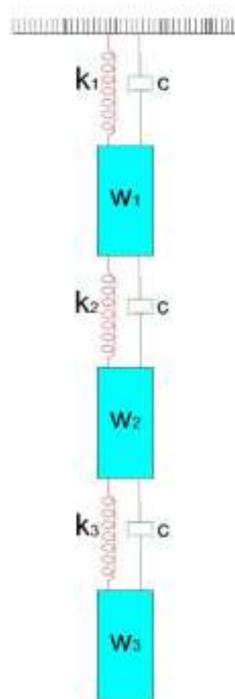


Figure 1: Idealized three degree of freedom system  
Degree of freedom

The weight  $w_1, w_2, w_3$  of this system is included with rigid mass which is suspended one after one to create triple pendulum. The elastic resistance to displacement is provided by the weightless ideal spring of stiffness  $k_1, k_2, k_3$  Figure 1: *Idealized three degree of freedom system Degree of freedom*. While the energy loss mechanism is represented by the damper  $c$ . If dynamic loading generated by the response of the structure is the time varying force  $p(t)$  and weight of pendulum is  $P$ , then Equation of motion of the basic dynamic system under influence of gravitational force is:

$$w \ddot{a}(t) + c \dot{a}(t) + k a(t) = p(t) + P$$

Mathematical model of a triple pendulum consists of three ( $i=1; 2; 3$ ) which are absolutely rigid bodies moving in a zero air resistance with a uniform gravitational field of lines that are parallel and directed against the axis  $X_2$  of the global coordinate system  $O_1X_1X_2X_3$ , connected to each other by means of revolute joints  $O_i$  and connected to a fixed base. Those joints have axes perpendicular to the plane  $O_1X_1X_2$  so that the whole system moves in planar motion. With assumptions that joints exist viscous damping, i.e. that the resistive moment counteracting the relative motion of two pendulums connected to each other is proportional (with a certain proportionality factor  $C_i$ ) to their relative angular velocity. Also assuming that mass centers of particular pendulums ( $C_i$ ) lie in planes determined by the axes of joints by which the given pendulum is connected to the rest of the system.

The last assumption allows for a decrease in the number of model parameters – to be precise, the number of parameters establishing the positions of mass centers of particular pendulums. Each of the pendulums has its own local coordinate system  $C_i Z_1^{(i)} Z_2^{(i)} Z_3^{(i)}$  ( $i=1,2,3$ ) of the origin at the mass center of the given pendulum and the axis  $Z_3^{(i)}$  perpendicular to the plane of motion. Geometrical constraints that determine the positions of mass Centres ( $e_i$ ) and the distances between the joints ( $d_1, d_2$ ) are indicated in Figure 2: *Triple physical*. Moreover, each of the pendulums possesses mass  $m_i$  and mass moment of inertia  $d_i$  with respect to the axis  $C_i Z_3$  passing through the mass center and perpendicular to the plane of motion. The first pendulum is acted upon by an external moment  $M_e(t)$ . The configuration of the system is uniquely described by three angles  $\theta_i$  as shown in **Error! Reference source not found.**

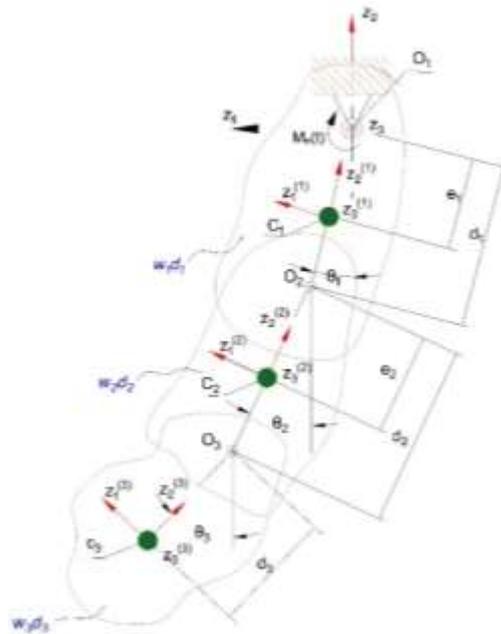


Figure 2: Triple physical Pendulum

Lagrange's equations of the second kind having the following form:

$$\frac{d}{dt} \left( \frac{dT}{dq_n} \right) - \frac{dT}{dq_n} + \frac{dV}{dq_n} = Q_n \quad n=1,2,3 \dots N$$

Where N is the number of generalized coordinates,  $q_n$  the nth generalized coordinate, T the kinetic energy of the system, V the potential energy, and  $Q_n$  the nth generalized force. For the triple pendulum, there are three angles of rotation  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  as generalized coordinates.

$$\frac{d}{dt} \left( \frac{dT}{d\theta_n} \right) - \frac{dT}{d\theta_n} + \frac{dV}{d\theta_n} = Q_n.$$

For a schematic representation of system refer **Error! Reference source not found.** Each bar  $i$  is defined by a set of four parameters:  $I_i$ , the moment of inertia of the bar,  $w_i$ , the mass of the bar,  $d_i$ , the length of the bar, and  $k_i$ , the damping coefficient of the bar rotating about its upper joint. The position and velocity of the bars are defined by the six system state variables:  $\theta_1 \theta_2 \theta_3 \dot{\theta}_1 \dot{\theta}_2 \dot{\theta}_3$ . Geometrical parameters of the system if the center of mass of the bars as function  $\theta_i$  and  $y$  is down,  $+x$  is right then :

$$y_1 = \frac{d_1}{2} \cos \theta_1$$

$$y_2 = d_1 \cos \theta_1 + \frac{d_2}{2} \cos \theta_2$$

$$y_3 = d_1 \cos \theta_1 + d_2 \cos \theta_2 + \frac{d_3}{2} \cos \theta_3$$

$$x_1 = \frac{d_1}{2} \sin \theta_1$$

$$x_2 = d_1 \sin \theta_1 + \frac{d_2}{2} \sin \theta_2$$

$$x_3 = d_1 \sin \theta_1 + d_2 \sin \theta_2 + \frac{d_3}{2} \sin \theta_3$$

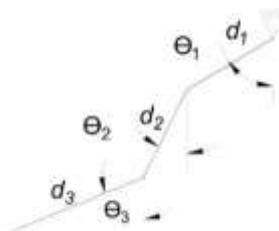


Figure 3: Schematic representation of the model

These positions are differentiated with respect to time to find x, y component of velocities of angles and angular velocities. The magnitude of velocity each pendulum bar:

$$v_i = \sqrt{\dot{x}_i^2} + \sqrt{\dot{y}_i^2}$$

The translational and rotational kinetic energy (TKE and RKE) of each pendulum bar:

$$TKE_i = \frac{1}{2} m_i v_i^2$$

$$RKE_i = \frac{1}{2} I_i \theta_i$$

The gravitational potential energy (GPE) of each pendulum bar is:

$$GPE_i = m_i g y_i$$

Using these, the Lagrangian of the system is,

$$L = T - V = \sum_{i=1}^3 TKE_i + RKE_i - GPE_i$$

Above equation can then be input to Lagrange's Equation.

$$\frac{d}{dt} \left( \frac{dL}{dq_i} \right) = \frac{dL}{dq_i}$$

Lagrange's Equation can be also rewritten as,

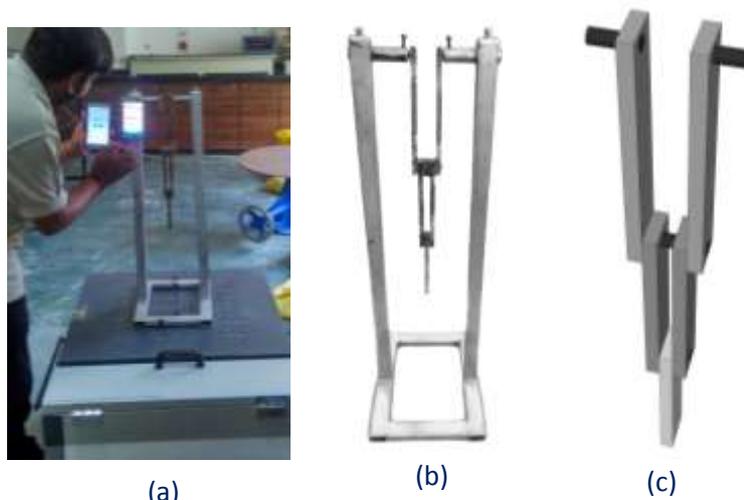
$$\frac{d}{dt} \left( \frac{dL}{dq_i} \right) = \frac{dL}{dq_i} - \frac{dD}{dq_i}$$

## 2. DYNAMIC RESPONSES OF TRIPLE PENDULUM

To study the dynamic behavior of a triple pendulum with experiments, a model of triple pendulum set on vibration table as shown in **Figure 4: (a) Experimental test setup (b) Model of triple pendulum (c) triple pendulum simulation model**. To validate motion of simulation, the same experiment is being carried out in computer-generated simulation and compared with experimental sensor data. The parameters of this experimental test setup were measured as shown in **Table 1: Mass and length of three pendulums**. and same values were used in the simulation.

**Table 1: Mass and length of three pendulums.**

Parameter	Values
m <sub>1</sub> (mass-1)	1200 grams
m <sub>2</sub> (mass-2)	850 grams
m <sub>3</sub> (mass-3)	400 grams
l <sub>1</sub> (length-1)	18.00 cm
l <sub>2</sub> (length-2)	15.00 cm
l <sub>3</sub> (length-3)	12.00 cm



**Figure 4: (a) Experimental test setup (b) Model of triple pendulum (c) triple pendulum simulation model**

In this research, triple pendulum has only one plane to oscillate i.e. displacement only in x and y-direction.

This type of two dimensional triple pendulum has eight physically possible positions as shown in fig x.x. from which stable condition is when all three masses are directed towards gravity.

### 3. FREE FALL OF PENDULUM

To observe hyperchaotic movement of the system, the triple pendulum is dropped with its self-weight toward gravity from a position of  $(\pi, \pi, \pi, 0, 0, 0)$ . Acceleration is measured in all three pendulums with help of sensor to plot the relation between each pendulum fig 7. Displacement in x and y-direction with respect to time is also measured with help of tracker. Tracking of each endpoint of the triple pendulum is being measured by long exposure camera which was same as simulation.

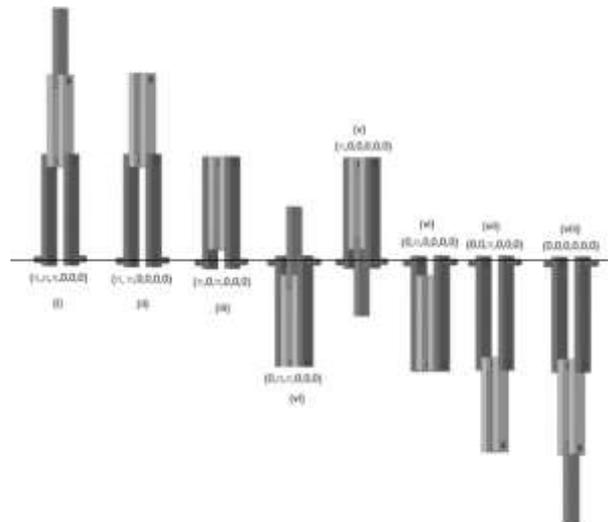
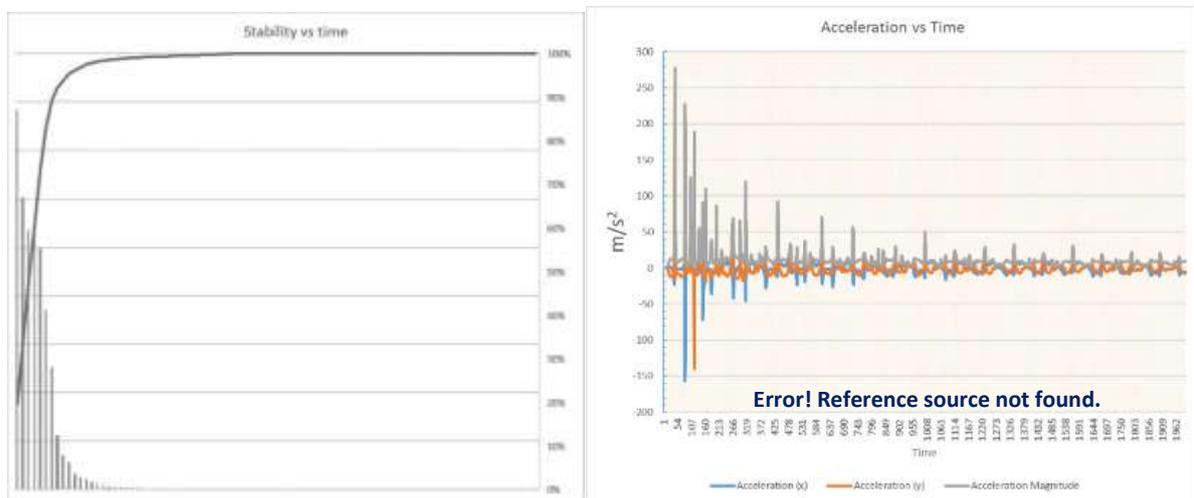


Figure 5 : The eight possible configurations of the pendulum. Unstable Conditions: (i) to (vii). Stable condition (viii).



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Figure 7: Acceleration (m/s<sup>2</sup>) vs Time (s)

To understand hyperchaotic behaviour of triple pendulum all possible sensor data were collected to plot data and inspect motion of the triple pendulum. Acceleration vs Time is plotted in Figure7. Velocity in Y-direction Vs Angular Momentum is plotted in Figure8-a; Velocity in X-direction Vs Angular momentum is plotted in Figure8-b; Velocity in X-direction Vs Acceleration magnitude is plotted in Figure8-c; Acceleration in Y-direction Vs Momentum in Y-direction is plotted in Figure8-d; Acceleration in X-direction Vs Momentum in X-direction is plotted in Figure8-e; Momentum in X-direction Vs Velocity in Y-direction is plotted in Figure8-f; Angular speed Vs Time is plotted in Figure8-g (h) Speed Vs Time is plotted in Figure8-h.

—3rd Pendulum —2nd Pendulum —1st Pendulum —3rd Pendulum —2nd Pendulum —1st Pendulum —3rd Pendulum —2nd Pendulum —1st Pendulum

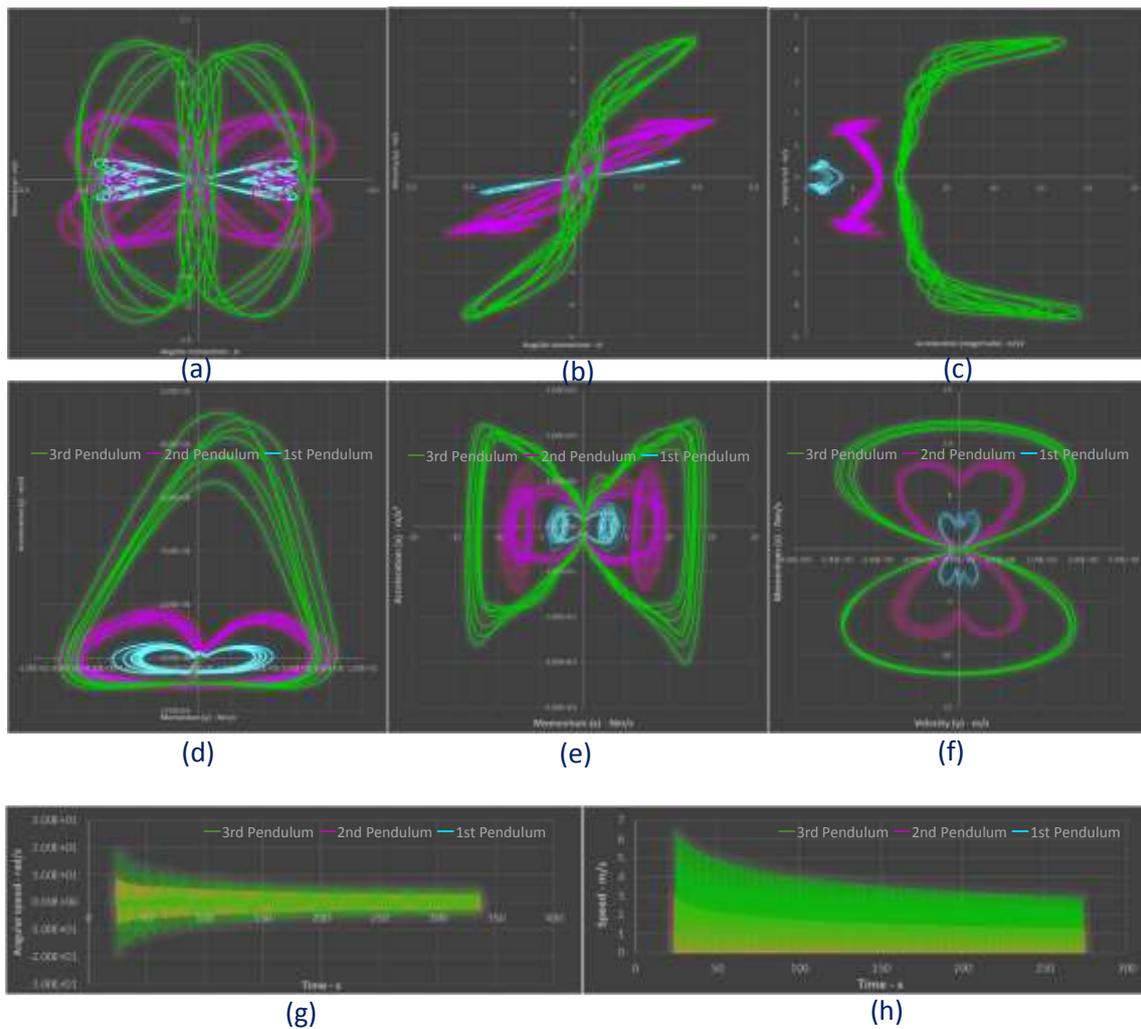


Figure 8: (a) Velocity-Y Vs Angular Momentum (b) Velocity-X Vs Angular momentum (c) Velocity-X Vs Acceleration (d) Acceleration-Y Vs Momentum-Y (e) Acceleration-X Vs Momentum-X (f) Momentum-X Vs Velocity-Y (g) Angular speed Vs Time (h) Speed Vs Time

### Parametric Study

A body without damping system, with single pendulum and with a triple pendulum is observed with a different frequency to observe the behaviour of the body under a different type of damping system with a pendulum. As shown in figure 9, triple pendulum was giving most effective damping on body compare to single pendulum system.

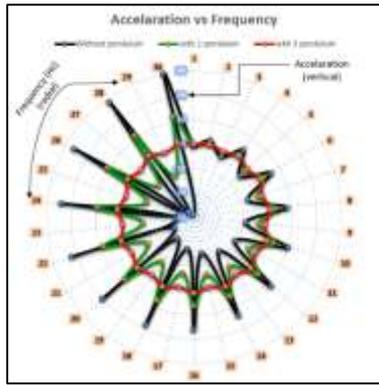


Figure 9: Acceleration vs Frequency with and without pendulum.



Figure 10: simulation model for floating structure

A structural arrangement as shown Fig 10 & 11 is tested for a single impact load in the system; Displacement in x directions are measured from simulation with the sensor and with the same amount of single impact force, computer-generated model is simulated to plot reading (fig 12) and match experimental data with simulation. The structural arrangement was tested for four conditions (i) No damping to structure (ii) structure with single pendulum (iii) structure with double pendulum (long-short) (iv) structure with double pendulum (short-long) (v) structure with a triple pendulum. For all these conditions weight of the structure is maintained same.

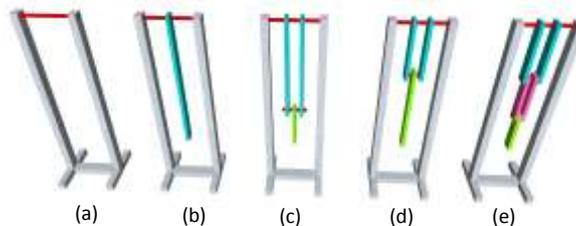


Figure 11: (a) No pendulum (b) single pendulum (c) double pendulum (long-short) (d) double pendulum (short-long) (e) Triple pendulum

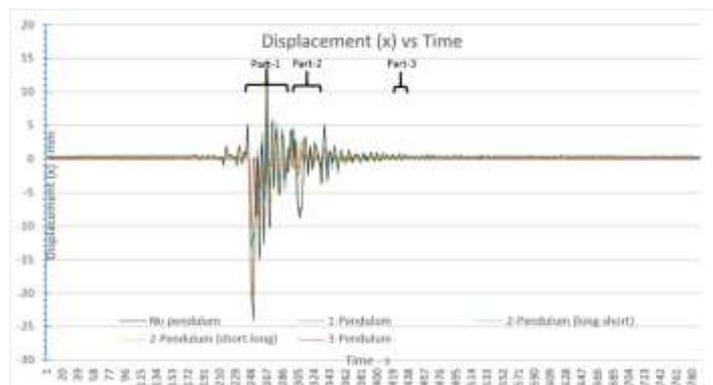


Figure 12: Displacement vs time

Displacement vs time plot for different conditions shows a change in structural behaviour as shown in fig 12. As shown in part-1 (fig.14a) starting point of initial impact force is same it is visibly clear that maximum displacement occurs in the structure without damping which is stated in a graph with a black line, the single pendulum is more effective than a double pendulum. The most effective system is with a triple pendulum as shown in fig 14b and fig 14c triple pendulum has minimum displacement and structure is stabilized in very less time compared to other structural systems.

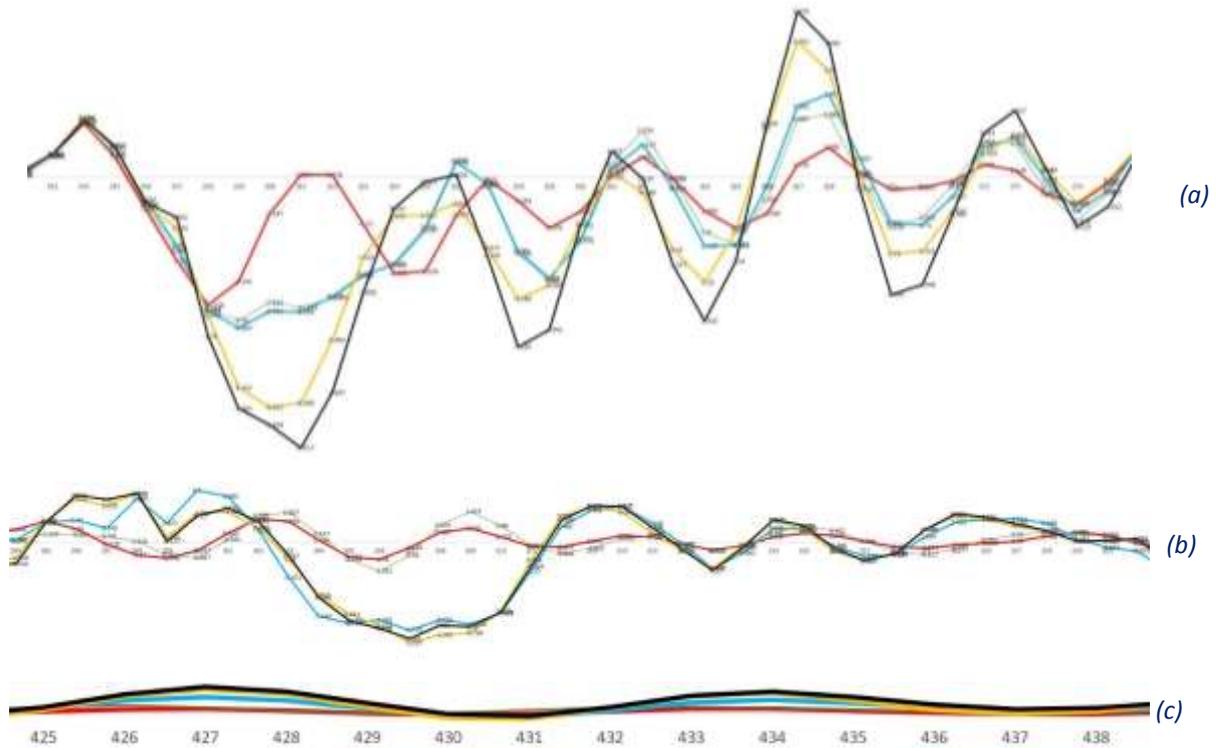
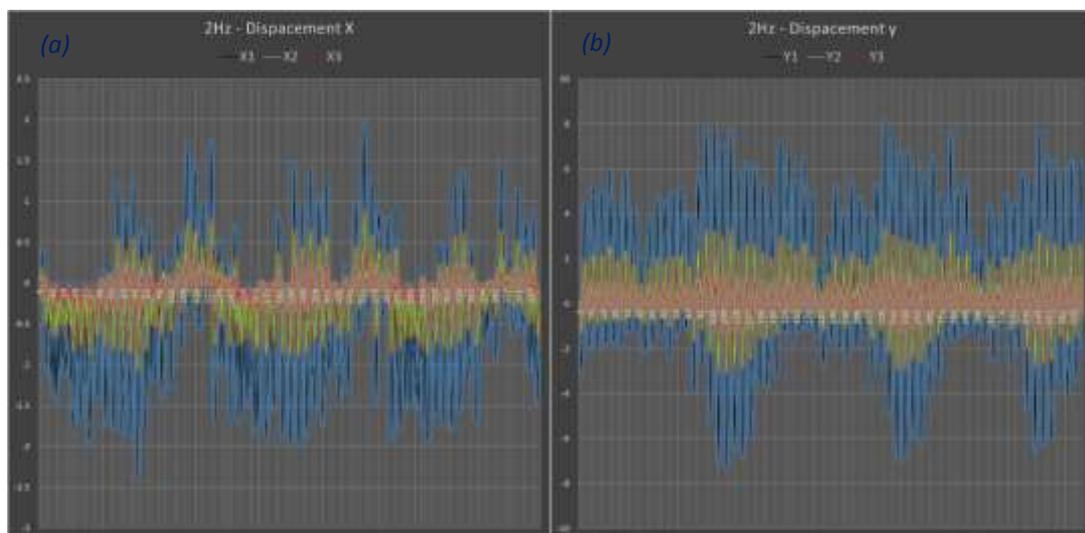
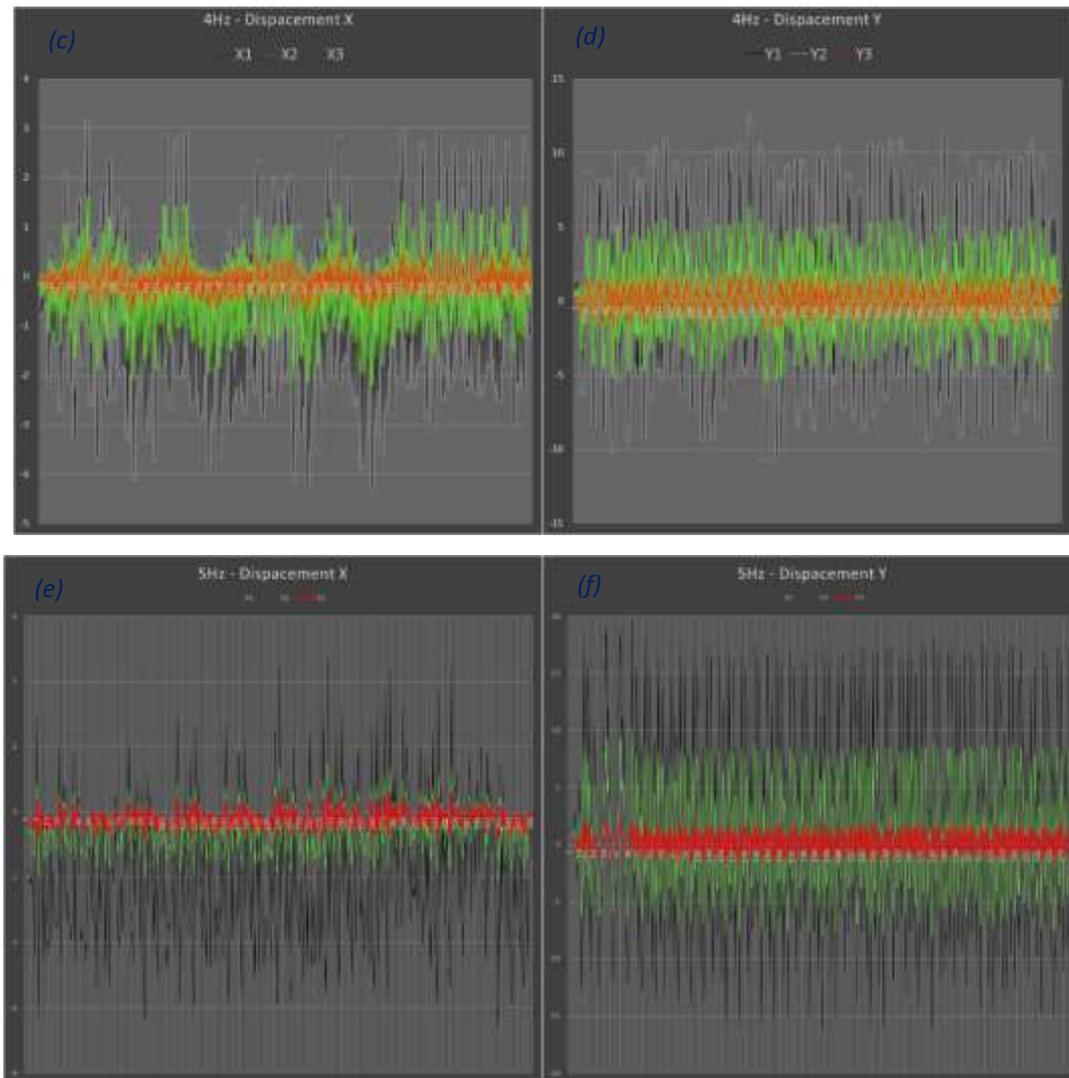


Figure 13: (a) Zoomed portion of fig.13 Part-1 (b) Zoomed portion of fig.13 Part-2 (c) Zoomed portion of fig.13 Part-3

It is clear that triple pendulum is much more effective compared to the single and double pendulum. To study the effectiveness of triple pendulum in the structural arrangement it is tested for different RPMs and sensor data is plotted as shown in fig 14.





**Figure 14: (a) For Frequency 2Hz, Displacement in X-Direction with Time (b) For Frequency 2Hz, Displacement in Y-Direction with Time (c) For Frequency 4Hz, Displacement in X-Direction with Time (d) For Frequency 4Hz, Displacement in Y-Direction with Time (e) For Frequency 5Hz, Displacement in X-Direction with Time (f) For Frequency 5Hz, Displacement in Y-Direction with Time**

It is observed that a floating structural system without damping has higher displacement rates in X and Y direction; by providing single pendulum as a damping solution reduces vibration up to 20% in the structural system. Using triple pendulum as a damping system in structural arrangement gives very small displacements in X and Y direction during vibration which are reduced up to 45% compared to the undamped structure. It is visually clear from charts of Figure 14 that triple pendulum is giving far more reduction in vibration control compare to single pendulum system.

## CONCLUSION

Study herein presented was aimed at implementing a passive control device to reduce vibration behaviour of any structure. With this concept in mind we can use the triple pendulum in floating bodies like ships, yachts etc. to stabilize the floating bodies while moving in water. The triple pendulum does not require any external energy source, as it develops control force from its relative motion of the mass, induced by the dynamic of the structure. Single and triple pendulum models have analysed with experiments as well as FEM based modeling and resulting dynamic behaviour of this type of bodies investigated. That is by proper weight-ratio & length of triple pendulum it is possible to control the lateral movement of bodies in water or to establish stabilization in ships while moving. The existence of triple pendulum system reduces lateral shaking, acceleration and force response of bodies that are subjected to vibration due to sea waves. The dynamic response of floating bodies without damping, single pendulum and with a triple pendulum was taken into consideration for study. Based on the result of this study a triple pendulum can provide much more effective lateral movement control compare to the single pendulum. The

triple pendulum can effectively control the shacking of floating bodies that induced by natural forces or artificial forces.

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